PBS Mathline®

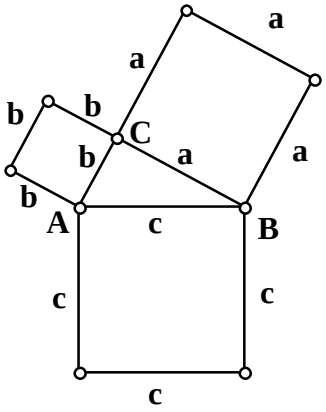
Activity 1:
Proving the Pythagorean Theorem

Pythagoras of Samos, c.560–480 BC, was a Greek philosopher and religious leader who was responsible for important developments in the history of mathematics, astronomy, and the theory of music. He migrated to Croton where he founded a philosophical and religious school that attracted many followers. Because no reliable contemporary records survive, and because the school practiced both secrecy and communalism, the contributions of Pythagoras himself and those of his followers cannot be distinguished. The most important discovery of this school was the fact that the diagonal of a square is not a rational multiple of its side (that is, the diagonal of a square is not a number that can be expressed as the ratio of two whole numbers.). In essence, this showed the existence of irrational numbers. This discovery disturbed Greek mathematicians and the Pythagoreans themselves, who believed that whole numbers and their ratios could account for geometrical properties. Pythagoreans believed that all relations could be reduced to number relations ("all things are numbers").

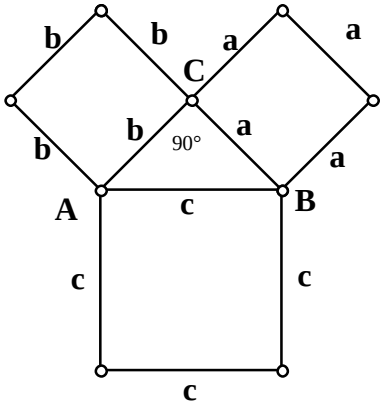
The Pythagoreans knew, as did the Egyptians before them, that any triangle whose sides were in the ratio 3:4:5 was a right-angled triangle. The so-called Pythagorean theorem, that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides, may have been known in Babylonia, where Pythagoras traveled in his youth. The Pythagoreans, however, are usually credited with the first proof of this theorem.

Much of the Pythagorean doctrine that has survived consists of numerology and number mysticism, and the influence of the belief that the world can be understood through mathematics. That belief was extremely important to the development of science and mathematics.

The following figure is the typical figure used to prove the Pythagorean theorem that the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. $c^2 = a^2 + b^2$

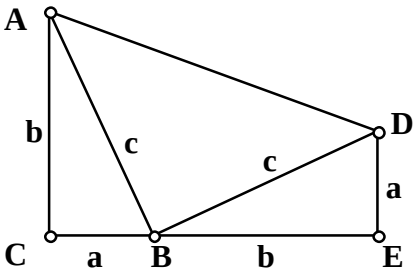


1. If the right triangle ACB were isosceles, the figure would appear as follows.



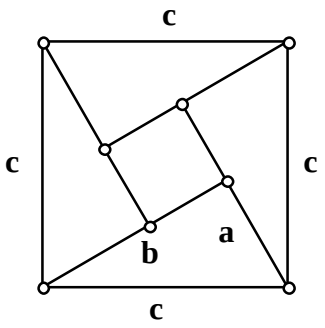
Show how with the addition of three lines the proof of the theorem is as simple as 2 + 2 = 4.

2. The following figure was used by President Garfield in proving the Pythagorean theorem. His method is based on the fact that the area of the trapezoid ACED is equal to the sum of the areas of the three right triangles ACB, ABD, and BED.



Prove the Pythagorean theorem using President Garfield's method.

3. A Hindu mathematician named Bhaskara used the following figure to prove the Pythagorean theorem by showing the sum of the area of the small square and the area of the four congruent right triangles equal the area of the large square.



Prove the Pythagorean theorem using Bhaskara's method.