



Activity 3: Pairwise Comparison

Student Page

For the Pairwise Comparison method, each candidate is matched head-to-head with each other candidate. Each candidate gets one point for a one-on-one win and a half point for a tie. The candidate with the most total points is the winner.

Look again at the election of the student council president used in the previous activities. Determine the winner this time using Pairwise Comparison. One thousand five hundred voters cast preference ballots summarized in the following preference schedule. The four candidates are: Gail, Twanda, Shawn, and Ricco.

	Number of votes received			
Place	390	360	300	450
1 st	Shawn	Gail	Gail	Ricco
2 nd	Twanda	Twanda	Twanda	Twanda
3 rd	Ricco	Ricco	Shawn	Shawn
4 th	Gail	Shawn	Ricco	Gail

The method of Pairwise Comparison revolves around head-to-head match-ups, so you need a way to organize and display all possible head-to-head match-ups. Consider the following table, in which each candidate has a row and column assigned.

	Shawn	Gail	Twanda	Ricco
Shawn				
Gail				
Twanda				
Ricco				

To use this table, select a head-to-head match-up, for example between Shawn and Gail. Go to the Shawn row and move over until you reach the column headed Gail, as shown by the * in the cell.

	Shawn	Gail	Twanda	Ricco
Shawn		*		
Gail				
Twanda				
Ricco				

Clearly, you will be duplicating your effort if you begin in the Gail row and move to the column headed Shawn, because that would also indicate a head-to-head match-up between the same two.

	Shawn	Gail	Twanda	Ricco
Shawn				
Gail	*			
Twanda				
Ricco				

To eliminate this duplication, cross out one of these cells for each candidate.

	Shawn	Gail	Twanda	Ricco
Shawn				
Gail	-----			
Twanda	-----	-----		
Ricco	-----	-----	-----	

Since no candidates will go head-to-head with themselves, you can also cross out the cells on the main diagonal.

	Shawn	Gail	Twanda	Ricco
Shawn	-----			
Gail	-----	-----		
Twanda	-----	-----	-----	
Ricco	-----	-----	-----	-----

The remaining 6 cells (because you have a four-candidate race) can be used to record the winner in all possible head-to-head match-ups.

1. Determine a formula for the number of cells in terms of the number of candidates N .

Now begin filling in the table of head-to-head match-ups for the student council election. To do this, you must go back to the preference schedule and determine how many voters prefer Shawn to Gail and how many prefer Gail to Shawn.

	Number of votes received			
Place	390	360	300	450
1 st	Shawn	Gail	Gail	Ricco
2 nd	Twanda	Twanda	Twanda	Twanda
3 rd	Ricco	Ricco	Shawn	Shawn
4 th	Gail	Shawn	Ricco	Gail

You can see that 840 voters prefer Shawn to Gail and 660 prefer Gail to Shawn. This indicates that in the head-to-head match-up, Shawn is the winner. Place Shawn in the cell indicating the winner.

	Shawn	Gail	Twanda	Ricco
Shawn	-----	Shawn		
Gail	-----	-----		
Twanda	-----	-----	-----	
Ricco	-----	-----	-----	-----

2. Who is the winner in the head-to-head match-up between Shawn and Twanda?

3. Who is the winner in the head-to-head match-up between Shawn and Ricco?

4. Who is the winner in the head-to-head match-up between Gail and Twanda?

5. Who is the winner in the head-to-head match-up between Gail and Ricco?

6. Who is the winner in the head-to-head match-up between Twanda and Ricco?

Now that we know the head-to-head winners, we can total all the scores and determine the winner of the election.

7. What is the point total for Shawn?

8. What is the point total for Gail?

9. What is the point total for Twanda?

10. What is the point total for Ricco?

Who wins the election using the Pairwise Comparison method?