PBS Mathline®

Algebraic Thinking Math Project

# Looking Through the Algebraic Lens
# Grades 3-5

## Overview

### Algebraic Thinking Focus

Teachers in grades 3-8 should make it a goal to foster the development of algebraic thinking in every strand of the mathematics curriculum. Topics in each strand offer opportunities to incorporate questions and activities that can lead to basic understanding needed for success in the formal study of algebra.

### Overview of the Lesson

This video has two parts. Part I, from the number and operations strand, shows students exploring adding and subtracting with odd and even numbers. Part II focuses on algebraic thinking drawn from a geometric setting. Students explore polygons using geoboards to discover a rule for the number of triangles inside polygons with different numbers of sides.

Part I: Looks Odd to Me

Part II: An Inside Look

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## Part I: Looks Odd to Me

### Lesson Objective

Students will work with rectangles to find generalizations for adding and subtracting odd and even numbers.

### Materials

For the class:

- Large chart paper for recording class findings

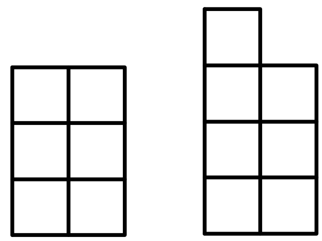
For each group:

- Paper rectangles
- Recording Sheets

## Procedure

**Prior to the Lesson**: It would be helpful, but not necessary, if students have some experience using "2 by" rectangles to represent odd and even numbers.

Examples:



6 = 2 by 3     7 = 2 by 3 plus 1

Thus, any even number can be represented by a "2 by" rectangle, a rectangle that is two squares wide. Any odd number is represented by a "2 by with a tail," a polygon that is two squares wide for every row except one which has only one square.

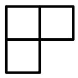
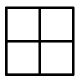
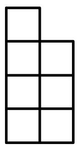
1. **Introduction**: Use the children's book *The Patchwork Quilt* by Valerie Flournoy to provide a meaningful context for even and odd numbers of squares. As explained by the children in the video classroom, at one point in the story an odd number of squares are available for a quilt and the quilt-maker must add one more square to make the quilt come out even. The children in the video classroom also made their own paper "2 by" quilt and discovered that they needed an even number of squares to make it a rectangle.

2. **Working with Models for Odd and Even Numbers**: Distribute some "2 by" and some "2 by with a tail" shapes to each small group of students. Have students identify some examples of shapes that represent odd numbers and some that represent even numbers. Tell them to work in their groups to find patterns when they add or subtract the numbers the shapes represent. Give each group recording sheets with the following headings:

| Illustrate Action | Words for Action | Numbers for Action | Math Language |
|---|---|---|---|
| | | | |

Discuss how students should record their findings.

Example:

| Illustrate Action | Words for Action | Numbers for Action | Math Language |
|---|---|---|---|
|  | 2 by with at tail | 3+4=7 | Odd + Even= Odd |
|  | 2 by 2 | | |
|  | 2 by 3 with a tail | | |

3. **Sharing Findings and Verifying the Generalizations:** Have groups share their findings and establish the generalizations for adding and subtracting odd and even numbers.

## Mathematically Speaking

***"Finding one's way around the number system and understanding its intricacies is an essential goal of elementary school mathematics. It is clearly a goal that transcends arithmetic. As students work with numbers, they also think about the characteristics of the numbers themselves. They describe, compare, and classify numbers. For example, a third grade class might investigate odd and even numbers. What makes a number odd? How do these numbers behave? When an odd number is added or subtracted from another odd number, the result is an even number. Why is this true?"***

Jan Mokros, et.al. in *Beyond Arithmetic*, 1995

One important way teachers can foster algebraic thinking is to challenge students to find patterns and make generalizations about numbers. Asking the students to use "2 by" and "2 by with a tail" shapes to explore adding and subtracting even and odd numbers is one such activity. In any formal course in algebra students will encounter problems that require them to apply understanding related to odd and even numbers. For example, they will be asked to add an odd number of negative numbers or raise a negative expression to an even power. Further, discovering that there are patterns in the results of adding and subtracting odd and even numbers helps students develop the habit of looking for generalizations. Developing these and other techniques as methods for understanding, students will learn to use mathematics to make sense of their world.

One important aspect of the lesson was the questioning technique used by the video teacher. She guided student thinking by posing the following questions:

- Tell me something that happened more than once.
- What would happen next time?
- Show me how that works using your shapes.
- Can you say that a different way?
- Can you show me another example of that idea?
- Let me hear you explain that to your group.
- Say that again using mathematical language.
- What would your rule predict?
- Ask someone else in your group to test your idea.

Students will begin to ask themselves questions like those posed in other problem situations as they become more independent learners.

Another important component of the lesson is the treatment of examples. First, students should be encouraged to try many problems before attempting to generalize. It should be made clear that one example does not prove a generalization. For example, a student might conclude that because 3 - 2 = 1, any time you take away (subtract) an even number the difference is odd. On the other hand, students should also recognize that one counter example disproves a generalization. So they should be encouraged to test any conjecture they have by trying to find a counter example. Similarly, they should be encouraged to use any possible generalization to predict what would happen for a new problem.

Using the "2 by" and "2 by with a tail" shapes, students have many opportunities to make discoveries beyond the objectives of this lesson. For example, one student in the video classroom proposed that an even number + an even number + an odd number = an odd number. The students in her group added many examples and could not provide a counter example. The group was not convinced that she was correct until she used the shapes to patiently explain her conclusion in the following steps: (1) We all agree even + even = even, (2) The even answer + odd = odd (since we all agree that even +odd = odd), and (3) If you rearrange the numbers to add even + odd first, you get odd and then add an even so your answer is still odd. Whether or not she knew the name for the associative property of addition, this student certainly utilized it in a convincing way. One group of students realized they could use the shapes to model multiplication. They clearly understood that multiplication is repeated addition and were probably ready to search for the generalizations in multiplying odd and even numbers. Finally, one student folded a "2 by" and noticed that he obtained an odd number. This discovery illustrates that the shapes could be used for exploring patterns in the division of numbers as well.

The whole-class discussion helps students see the discoveries of others. It also provides the "big picture" of the patterns that exist with adding and subtracting odd and even numbers for students who may need to hear them again. It is important that students get the opportunity to synthesize all the generalizations at the end of the activity.

Finally, an important observation was made by one of the students during the lesson. He observed that while the class had provided three rules for adding odd and even numbers (even + even, odd + odd, and even + odd), there are actually four cases: even + odd, he pointed out, is different than odd + even. Attention to order in a situation and whether it makes a difference is an important concept. While order does not matter in addition, it certainly does in subtraction-despite the fact that both cases in this situation yield the same result.

## Extension

- The children's book, *One Hundred Hungry Ants* by Elinor Pinczes can be used to introduce the use of rectangular arrays to explore patterns with multiplication of odd and even numbers. In the book, 100 ants first march in a column 1 wide by 100 long (1 x 100 or **odd x even**). They then march in 2 columns each 50 long (2 x 50 or **even x even**), followed by 4 columns of 25 (**even x odd**), 5 columns of 20 (**odd x even**) and 10 columns of 10(**even x even**). For every array the ants can make, of course, the result is even (100). Provide the students with one inch grid paper and scissors. Then students can cut as many rectangular arrays as they can find for 10 ants, 15 ants, 25 ants, etc. to find the generalizations for multiplying odd and even numbers.

  They should discover the following rules:
  Odd x Even = Even
  Even x Odd = Even
  Even x Even = Even
  Odd x Odd = Odd

  This activity also relates to finding all the factors for a number. Only those numbers that can be used as the length or width of a rectangular array containing the same number of squares as the value of a number are factors of that number.

## Resources

### Books:

❖Fluornoy, Valerie. The Patchwork Quilt. New York: EP Dutton, Inc., 1952.

❖Mokros, Jan, Susan Jo Russell, and Karen Economopoulos. Beyond Arithmetic. Palo Alto: Dale Seymour Publications, 1995.

❖Pinczes, Elinor. One Hundred Hungry Ants. Boston: Houghton Mifflin, 1993.

**Other:**

❖*Principles and Standards of School Mathematics:* (Draft), National Council of Teachers of Mathematics, Reston, VA: NCTM, 1998.
*http://www.nctm.org*

## Algebraic Thinking Math Project

# Looking Through the Algebraic Lens
# Grades 3-5

## Part 2: An Inside Look

### Lesson Objective

Students use geoboards to construct polygons, look for patterns, and find a general rule.

### Materials

For the class:
- Large chart paper for recording class findings

For each student:
- Geoboards
- Rubber Bands
- Geodot Paper

### Procedure

**Prior to the Lesson**: Students should be able to recognize whether or not a figure is a polygon. They should know that a polygon has more than two straight sides, is a closed figure, and has no intersections except at the vertices. The teacher in the video classroom used The Greedy Triangle by Marilyn Burns to introduce her students to the topic prior to this lesson.

1. **Introduction**: Review the properties of polygons by playing a sorting game. Have the students sort shapes into two groups, those that are polygons and those that are not. Use the game to elicit from students the properties of a polygon.

2. **Working with Pentagons**: Distribute geoboards and have each student construct a five-sided polygon (pentagon). Verify that all students were successful. Then tell them to divide the pentagon into triangular regions. Stress that each triangle must start at the same vertex of the pentagon and must not overlap any other triangle. Have students predict how many triangles they might find. When all students are finished, have them share their findings and summarize the overall result. They should conclude that inside every pentagon are exactly three triangles.

3. **Working with other Polygons**: Have students work in small groups to explore triangles inside different polygons (triangle, quadrilateral, pentagon, hexagon, septagon, octagon, nonogon, and decagon). Have each group create several polygons on their geoboards. Using rubber bands, students create triangles adhering to the rules that each triangle must start at the same vertex and not intersect with another triangle. After creating triangles on the geoboards students sketch their constructions on geodot paper. Tell the students to look for a pattern concerning the number of possible triangles inside a polygon.

4. **Sharing Findings and Verifying the Generalization**: Have students view the findings and record an arrangement of each group on their geodot paper. They should record the number of sides and the number of triangles in each polygon. Lead a class discussion about the findings and the generalizations. Students should conclude that the number of possible triangles is always two fewer than the number of sides.

## Mathematically Speaking

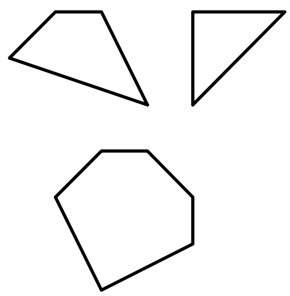
***"Students in grades 3-5 are encouraged to look for patterns and to express them mathematically (in words or symbols) in order to help them discover, understand, and use patterns of algebra to solve a variety of problems….Representing a pattern both geometrically and numerically pushes students in their understanding of the situation and provides multiple perspectives of patterns and relationships."***

*Principles and Standards for School Mathematics* (Draft), 1998

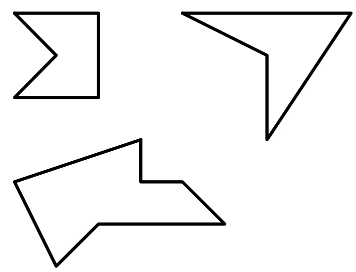
Many topics in geometry offer contexts for students to combine spatial visualization with numerical patterns. Making connections between shapes and numbers help students see the relationships between mathematical strands and the power of multiple representations. Discovering that there are patterns in dealing with polygons also helps students develop the habit of looking for and generalizing patterns.

It is important that students understand the properties of a polygon before attempting this lesson. They should know that every polygon is a closed figure which has more than two sides made of line segments which do not intersect except at their endpoints. As pointed out in the *NCTM Principles and Standards for School Mathematics,* "In primary grades students…begin to develop and use a basic vocabulary related to these shapes but do not develop precise meanings for many of the terms they use." To be successful with this lesson, students need only be able to identify and construct polygons. It should be noted that students in the video class had a lot of difficulty constructing triangles inside concave polygons, so the teacher should limit the students to convex polygons: third graders do not have difficulty understanding that a convex polygon cannot "jut in" or "have a dent" as shown below. However, concave polygons can be formed on the geoboard, but should not be considered at this time.

Convex Polygons



Concave Polygons



Some students were distracted by the pegs on the geoboard that were inside the polygon they constructed. In fact, several students tried to use interior points as vertices of their triangles. The teacher should emphasize that the vertices of each triangle must also be vertices of the original polygon. To help remedy this problem students can place a stick-on dot on top of each peg that serves as a vertex in the polygon or the teacher may instruct students to use a geoboard only a circle of pegs.

*Using the geoboard with only the circle of pegs prevents stduents from constructing concave polygons. However, this restriction decreases the total number of possible convex polygons that can be constructed.*

One group should investigate creating triangles inside three-sided polygons. It is important that students discover that there can be only the original triangle with three-sided polygons and include this information in their table of findings, as shown below:

| Name of Polygon | Number of Sides | Number of Triangles |
|---|---|---|
| Triangle | 3 | 1 |
| Quadrilateral | 4 | 2 |
| Pentagon | 5 | 3 |
| Hexagon | 6 | 4 |
| Septagon | 7 | 5 |
| Octagon | 8 | 6 |
| Nonagon | 9 | 7 |
| Decagon | 10 | 8 |

Again note that the vocabulary (e.g., mathematical name for each kind of polygon) is not the emphasis of this lesson. Instead, having students find the relationship between the number of sides in the polygon and the number of interior triangles that can be drawn from any vertex is the objective.

It was important that students are encouraged to express the general rule in their own words. For example, in the video class, one student (Charlie) stated, "The number of sides is two more than the number of triangles," while most students seemed to be more comfortable with, "The number of triangles is always two less than the number of sides."

In this particular lesson, since you always start with the polygon and know the number of sides, it is natural that most students are more comfortable with the second form of the rule and you should make sure that form of the rule is expressed. In general, encouraging students to word a generalization in a way that makes sense to them, *as long as the correct meaning is there,* is not only valuable for the specific lesson but helps children to understand that there is often more than one correct way to express a mathematical relationship.

Finally, this lesson lays the foundation for students to later discover the total number of degrees in the interior angles or the total number of diagonals of any polygon. While those activities may not be experienced until several years later, some students may remember dividing polygons into interior triangles. Thus, the impact of this lesson is multiple.

## Extension

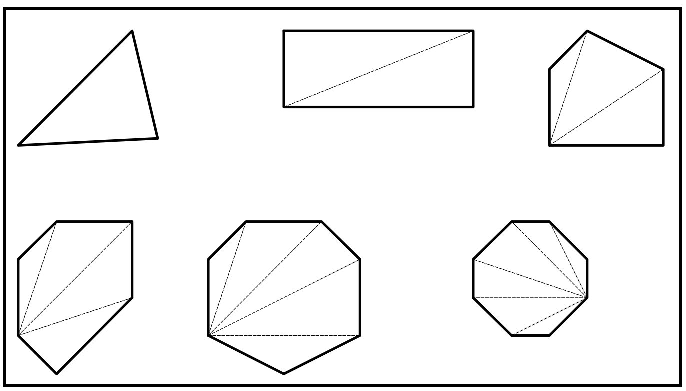
- Students can write a summary of the activity. They might be given any or all of the following guidelines:
- Include drawings of several polygons with different numbers of sides, each accompanied by a written explanation.
- Include a drawing or table of the pattern that the class discovered and an explanation. (Headings for the columns in the table might be given.)
- Use as much mathematical language as you can.
- Describe or draw a polygon with at least 7 sides. Predict how many triangles would be inside that polygon. Explain your thinking.

The teacher can use the summaries to assess student understanding of the concepts and their comfort with mathematical terms such as the names for various polygons.

## Technology Connections

- Have students use a dynamic software program to create polygons and the triangles inside those polygons. Once the polygons and triangles are constructed, the students can explain orally or in writing the answer the following question: How does your sketch provide examples of the pattern we found about the number of triangles possible inside polygons?

  Example:



  Other questions students might investigate, if appropriate, including the following:
  - Can you draw the triangles inside a polygon from any vertex?
  - Does it make a difference which way the polygon is turned?
  - Are the areas of all the inside triangles the same?

- Visit *http://www.mathclub.com/cgi-dat/e_geoboard.pl* and download an electronic geoboard for free.

## Resources

### Books:

❖Burns, Marilyn. The Greedy Triangle. New York: Scholastic Press, 1994.

### Other:

❖*Principles and Standards of School Mathematics* (Draft),National Council of Teachers of Mathematics, Reston, VA: NCTM, 1998.
*http://www.nctm.org*

❖Geoboards:
*http://www.cuisenaire-dsp.com/ss-geoboards.html*

# Other MATHLINE Lessons that Provide Algebraic Thinking Across the Mathematics Curriculum in Grades 3-8

The following lessons are found in earlier MATHLINE projects, either The Elementary School Math Project (ESMP) or the Middle School Math Project (MSMP). The videos and support materials can be obtained through PBS. For some lessons you will need to make adaptations to emphasize the development of algebraic thinking.

| Strand | Project | Lesson Title | Description | Connection |
|---|---|---|---|---|
| Measurement | ESMP | Bubble Mania | Measuring the diameter, circumference, and area of circles made by a bubble print | Deriving algebraic expressions and formulas |
| Number and Operations | ESMP | It Takes Ten | Estimating and measuring through a variety of lab experiences | Proportional Reasoning |
| Number and Operations | ESMP | Soak It Up | Comparing products to determine the best value | Proportional Reasoning, Unit Rates |
| Number and Operations | ESMP | An Apple a Day | Making estimates to analyze the number of apples per acre and visualizing the magnitude of one million | Proportional Reasoning, Unit Rates |
| Number and Operations | ESMP | Food for Thought | Identifying and comparing unit costs of given items | Proportional Reasoning, Unit Rates |
| Patterns, Functions, and Algebra | ESMP | Peddling Petals | Creating, extending, and describing arithmetic patterns found in paper flower designs | Patterns |
| Patterns, Functions, and Algebra | ESMP | Struts n' Stuff | Identifying the relationship between the number of sides in a regular polygon and the number of struts needed to make each polygon rigid | Patterns and Variables |
| Patterns, Functions, and Algebra | MSMP | Aw Chute | Determining and comparing the rate of descent of various student constructed parachutes | Proportional Reasoning, Rate |
| Patterns, Functions, and Algebra | MSMP | In a Heartbeat | Using scatterplots to determine the correlation between heartbeats per minute before and after aerobic exercise | Proportional Reasoning, Rate |
| Geometry | MSMP | Let's Face It | Identifying, describing, and constructing the five regular polyhedral | Patterns and Variables |
| Measurement | MSMP | How Many Noses Are In Your Arm | Applying proportional reasoning to determine the length of the Statue of Liberty's torch-bearing arm | Proportional Reasoning |
| Patterns, Functions, and Algebra | MSMP | Fill 'er Up | Interpreting, predicting, and sketching graphs of related functions as applied to the shapes of bottles | Functions |
| Patterns, Functions, and Algebra | MSMP | The Great Race | Constructing number patterns generated from a tortoise and the hare race and using them to generate a graph of the situation | Patterns and Functions |
| Statistics | MSMP | Something Fishy | Applying proportional reasoning to the capture-recapture statistical procedure | Proportional Reasoning |

## Ideas for Online Discussion

Ideas are linked to the Principles and Standards for School Mathematics.

1. Give a specific example of a lesson traditionally categorized in the number and operations strand that you have used in your classroom with an emphasis on the development of algebraic thinking.

2. Give a specific example of a lesson traditionally categorized in the geometry strand that you have used in your classroom with an emphasis on the development of algebraic thinking.

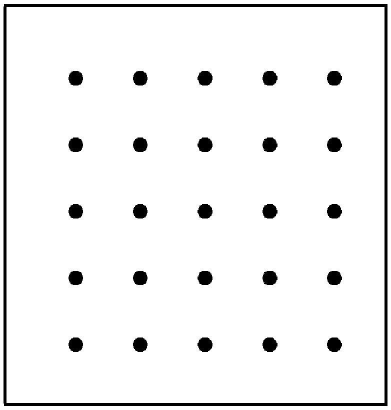
3. What are some opportunities in problem-solving activities at your grade level(s) to promote the development of algebraic thinking across the strands of the mathematics curriculum?

4. What effective ways have you found to link mathematics and children's literature in your classroom?

5. Which strands of the mathematics curriculum do you find it especially challenging to incorporate an emphasis on the development of algebraic thinking? Why do you think this is true?

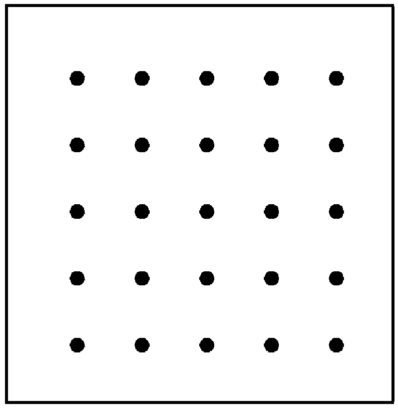
Name:___________________
Date:___________

# GEODOTS



Number of Sides:_____

Number of Triangles:___



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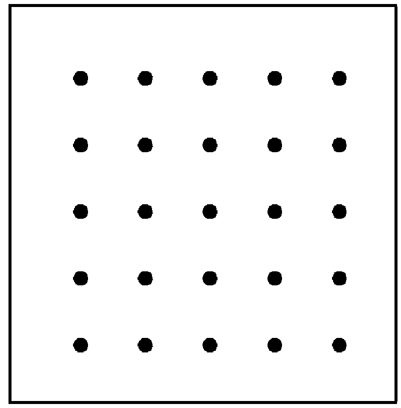
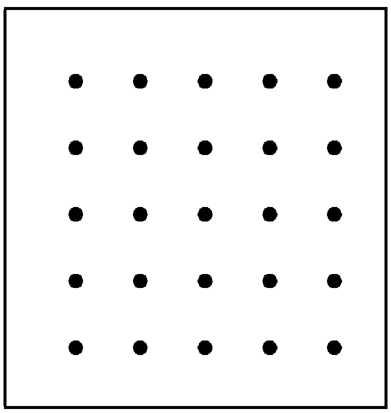


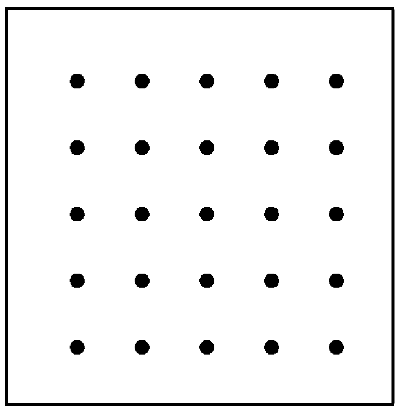
Number of Sides:_____

Number of Triangles:___



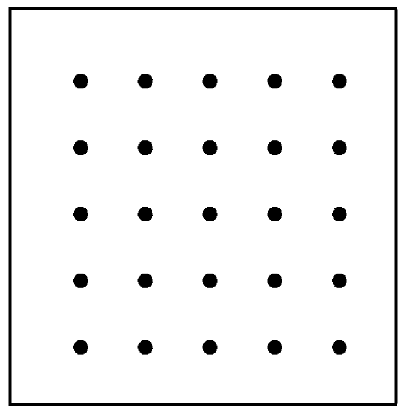
Number of Sides:_____

Number of Triangles:___



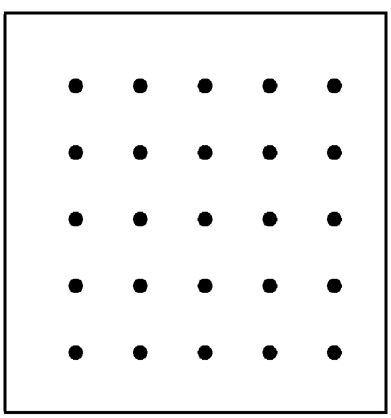
Number of Sides:_____

Number of Triangles:___



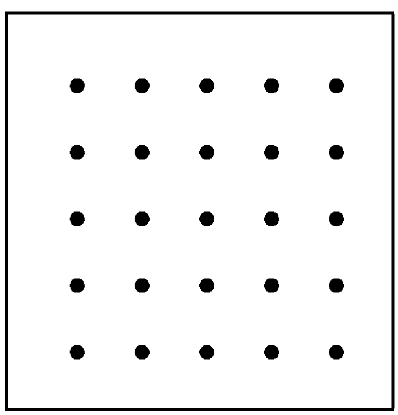
Number of Sides:_____

Number of Triangles:___



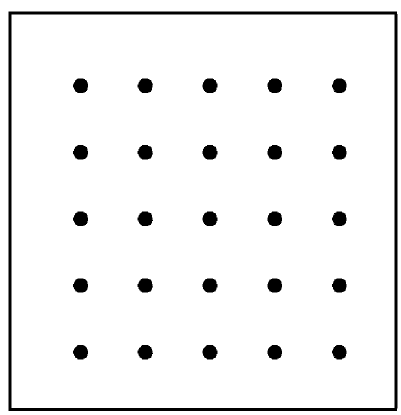
Number of Sides:_____

Number of Triangles:___



Number of Sides:_____

Number of Triangles:___



Number of Sides:_____

Number of Triangles:___