

## Algebraic Thinking Math Project

# You Make the Call

### Algebraic Thinking Focus

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The opportunity to examine and understand linear functions is a key component to success in the formal study of algebra. Moreover, students need to be able to describe relationships with a variety of representations and connect these representations—precisely as demonstrated by this lesson. These experiences foster the development of algebraic thinking in a curriculum, which is based on problem solving.

### Lesson Objective

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Students will find the function rule for each of two sets of data and compare the results to solve a problem.

### Overview of the Lesson

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Students are given two plans for cell phones. The first plan is given in the form of a table and the second in a graph. Students will determine the better plan for Scott, a middle school student, and explain their reasoning.

### Materials

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For each group:

- *Cell Phones Unlimited* and *A Phone Plan For You* activity sheets
- Chart paper
- Markers
- Rulers or yard/meter sticks

## Procedure

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**Prior to the Lesson:** Students should have prior experience with interpreting tables and graphs. They should also have worked with finding a function rule for a set of data. It would be helpful, but not necessary, if they have worked with problems that require comparisons of sets of data.

1. **Introduction:** Present students with a middle school student's dilemma. The student, Scott, must decide between two cell phone plans. Let the students briefly discuss the situation.
2. **Examining Data in a Table:** Show students the first plan, *A Phone Plan For You*, which is presented in table form.

*See the advertisement activity sheet at the end of this lesson.*

After identifying the basic information provided by the table, form groups and instruct students to analyze the plan and be prepared to explain how it works to the rest of the class. Allow the groups time to interpret the plan. Have chart paper and markers available. Instruct students to use these materials to record the information they extract from the table, any patterns they find, and a general rule (if they determine one).

3. **Examining Data in Graphic Form and Comparing Plans:** Introduce students to the second plan, *Cell Phones Unlimited*, which is presented in graphic form.

*See the sample mock advertisement at the end of this lesson.*

Again, students should be instructed to interpret the second plan in their groups. Then, each group should choose a method to compare the two plans and be prepared to present and explain their reasoning to the class.

4. **Group Presentations:** When all groups have finished, display the charts and instruct each group to explain the strategies they used and their results. Compare alternate forms of function rules, address any incorrect mathematics in the solutions, and encourage discussion if groups disagree on the better plan.

*Examples of student charts are at the end of this lesson.*

5. **Conclusion:** Finally, ask students to use their rules to determine the monthly cost if Scott talked 300 minutes in one month under each plan. Compare results calculated by using various forms of the rules to verify for students that the rules are equivalent. Ask students to address whether the results support their decision about which plan is better and ask them to explain their reasoning.

## Mathematically Speaking

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***“If mathematical concepts are developed from problem situations or contexts, then the variables in the situation and the relationship between them become real to the student. The problem context can provide the scaffolding needed to reason about the quantities in the situation and how they change in relationship to each other...Over time, through such experiences, students develop a rich language, moving freely among the different forms of representations, pictures, words, tables, graphs, and symbols...The understanding needed to move freely among different representations takes time to develop and hence is an important part of the curriculum throughout.”***

Elizabeth Phillips in *The Nature and Role of Algebra in the K-14 Curriculum*, 1998

Students are familiar with the topic of cell phones and the problem provides data in a context which has meaning for students at this level. When students interpret data in a problem solving situation, concepts related to variables are easier to grasp and to retain. For example, in this lesson, it is easy for most students to understand that the monthly cost for a plan (dependent variable) changed according to the number of minutes of service (independent variable). The fixed monthly rate also provided at least an intuitive introduction to y-intercept: \$9.95 is the monthly cost (function value) when the number of minutes (x value) was zero. Finally, students found that the point (100, ~35), indicated the number of minutes for which the two plans cost the same and was the intersection of the two graphs. The wider the variety of good problem solving experiences students have, the more meaning they are able to attach to mathematical terms and the more likely they are to be successful within a new context.

Another important element of this lesson is the exposure to multiple representations. Using one plan (*A Phone Plan For You*) presented data in a table and the other (*Cell Phones Unlimited*) presented in a graph obviously offers the opportunity for experience with both formats. Further, allowing students to choose either format for comparing the two plans lets them work with the representation with which they feel most comfortable while the small group presentations give them exposure to the alternate formats. Over time, this exposure will help students develop comfort and facility with all the forms of representation.

For some groups of students, this may be the first time they encounter a problem situation in which there is not a clear-cut right answer. While the *Phone Plan For You* costs less if Scott talked from 0 to 100 minutes, the *Cell Phones Unlimited* plan costs less if Scott talked more than 100 minutes. Students will usually agree that Scott most likely cannot afford to pay for more than 100 minutes anyway, so he should go with the *Phone Plan For You*. It is valuable for students to grapple with situations where several factors contribute to determining the best solution and one right answer is not instantly obvious, just as things are in the real world.

Alternate forms of function rules may be derived by your students. For example, a

group of students may express the rule for *A Phone Plan For You* as  $\$9.95 + \$0.25x$  where  $x$  = total number of minutes talked. Another group may come up with  $\$2.50(x/10) + \$9.95$  with  $x$  again = total minutes talked for the month. If this happens with your class, as in the video classroom, it is crucial that students understand that all the correct rules for the plans are equivalent. Most students will be convinced that this is true by comparing the table of values generated for each expression (either by hand and/or using the calculator). When the function values are the same for every value of  $x$ , many students become confident that the rules are equivalent. However, there are numerous students who need to evaluate each expression for one or more values of  $x$  before they are convinced. In such cases, a different student can be designated to evaluate each suggested form of the function rule at the board or overhead by substituting a particular value (10, e.g.) for each  $x$  in the rule his/her group derived. The function values can be compared and the process can be repeated for other values of  $x$ , if needed. For still other students, seeing that the graphs for all the equivalent expressions are identical is the most convincing approach (see Extension below).

Another topic related to this lesson is discrete versus continuous functions. In *A Phone Plan For You*, monthly costs are listed for 0, 10, 20, 30, etc., minutes. Some students may wonder what the monthly cost might be for times not listed, e.g., 15 minutes. Students may debate how the company charges the customer. Some companies charge by the minute and others by increments of minutes. These questions, while unanswered by the advertisement, provide a springboard for a discussion of discrete and continuous functions. Students at this level often have only dealt with continuous functions for which the coordinates are all connected. For discrete functions there is no such connected graph because there are no function values between the plotted coordinates.

Students need numerous experiences in rich problem-solving contexts to foster algebraic thinking. Further, they need to become comfortable with various forms of representation—both to interpret data and also to compare it in different forms. These are essential components of a powerful, practical approach to the teaching and learning of mathematics.

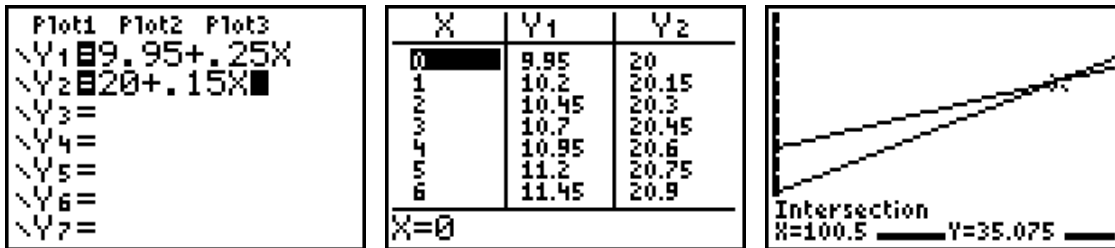
## **Extensions**

- You may find it appropriate to add a third cell phone plan and ask students to re-examine their recommendation for Scott. For example, the plan Yaks Unlimited offers a flat fee of \$28 for up to 100 minutes of air time and \$.30 for each additional minute. Such an extension offers an opportunity for students who struggled with the lesson a chance to reapply the skills immediately. The addition of a third plan also complicates the decision of which plan to recommend.
- Students can research cell phone plans in your area and compare them in a way similar to the one shown in the lesson. You may need to establish the type of cell

phone plan students should compare since companies offer a variety of plans which may cause problems for a direct comparison.



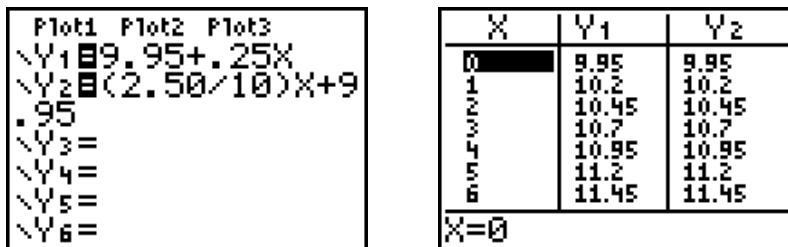
- Using graphing calculators such as the TI 73/82/83, it is effective to have students enter the rules for the plans in the Y= menu and view the results in a table and on a graph, as shown below. Using the TRACE feature can also pinpoint the intersection of the graphs and lead to a discussion about the meaning of that point.



Detailed instructions for building tables, setting graphing windows, and graphing functions are available for each TI graphing calculator model. Use the Quick Search option on the TI home page ([www.ti.com](http://www.ti.com)) to locate the appropriate guide.

- If students derive different versions of the rule for a particular plan, they can enter the various forms of the general rule in the Y = menu and view the results in a table. Seeing that the expressions all produce the same values using this quick, visual technique is an impressive way to reinforce that all their expressions are equivalent.

Example:



This can be followed by viewing the graph for each version and demonstrating that they are concurrent.

- For online research about mobile phones, your students can visit several mobile phone web sites. Wireless Dimension, <http://www.wirelessdimension.com> lets users in the top 50 US cities compare many of the available wireless phones and service plans in the area.

- To see how to graph more complicated phone plans as piecewise functions, visit <http://www.ti.com/calc/docs/act/bailey01.htm>

## Resources

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### Articles:

- ❖ Curcio, Fran, et al. “Exploring Patterns in Nonroutine Problems.” \_\_\_\_\_  
Teaching in The Middle School. February 1997, 262-269.
- Phillips, Elizabeth. “Developing a Coherent and Focused K-14 Algebra Curriculum.”  
The Nature and Role of Algebra in the K-14 Curriculum Washington, DC:  
National Academy Press, 1998.

### Other:

- ❖ \_\_\_\_\_ (Draft),  
National Council of Teachers of Mathematics (NCTM)  
<http://www.nctm.org>
- Texas Instruments Incorporated  
<http://www.ti.com>

## Ideas for Online Discussion

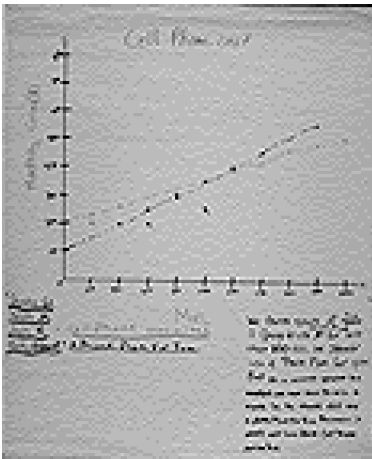
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1. A focus of this lesson is multiple representations of data in tables and graphs. This the benefits for students? Why or why not?
2. Often students are uncomfortable with problems that do not have one clear-cut,
3. When students deal with real world data, the numbers are often “messy.” What effects does this factor have on student learning?  
  
when they work in groups?
5. How can experiences with mathematically rich problems be structured to maximize

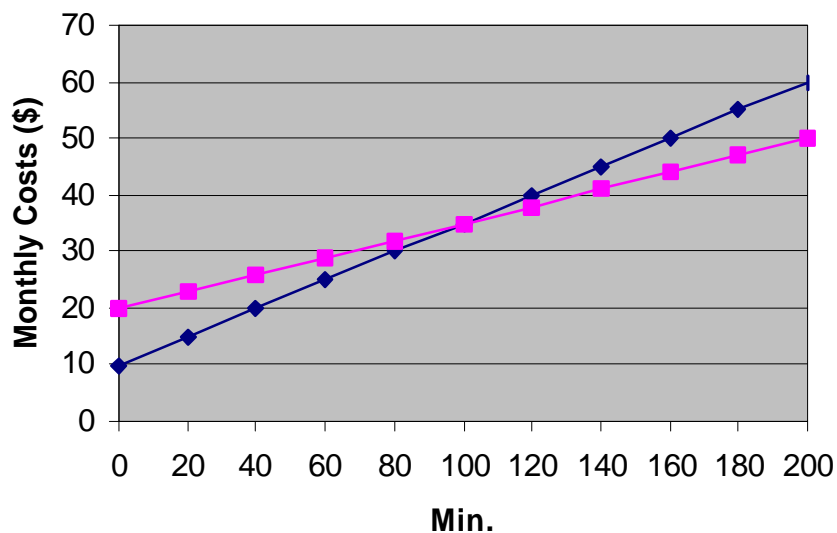
# Example of Student Work

## Group 1

Shana W.  
Kevin M.  
Zach R.  
Kelly O.



### Cell Phone Cost



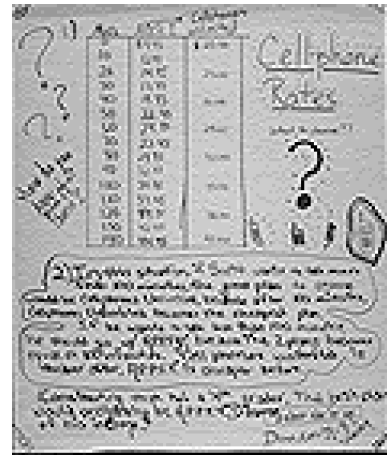
- Cell Phone Unlimited
- ◆ A Phone Plan For You

We think that if he is going to use the cell phone for less than 100 min. he should use A Phone Plan for you. But as a seventh grader he wouldn't use more than 80 minutes a month. so he should still use a phone Plan For You. Because it would cost less than Cell Phone Unlimited.

Example of Student Work

# Group 2

Drew  
Emily  
D.J.  
Steven



1. Cell Phones

Min.	A.P.P.F.Y.	Unlimited
0	\$9.95	\$20.00
10	12.45	
20	14.95	23.00
30	17.45	
40	19.95	26.00
50	22.45	
60	24.95	29.00
70	27.45	
80	29.95	32.00
90	32.45	
100	34.95	35.00
110	37.45	
120	39.95	38.00
130	42.45	
140	44.95	41.00

## Cell-phone Rates

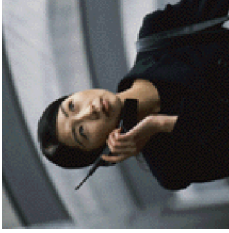
What to choose??

In this situation, if Scott wants to talk more than 100 minutes, the good plan to choose would be Cell phones Unlimited, because after 100 minutes, Cell phones Unlimited becomes the cheapest plan. If he wants to talk less than 100 minutes, he should go w/ (A.P.P.F.Y.)\*, because the 2 plans become equal at 100 minutes. Cell phones Unlimited is cheaper after, A.P.P.F.Y.\* is cheaper before.

(Considering that he's a 7<sup>th</sup> grader, the best plan would probably be A.P.P.F.Y.\*,) because of his income.)

\*A Phone Plan For You

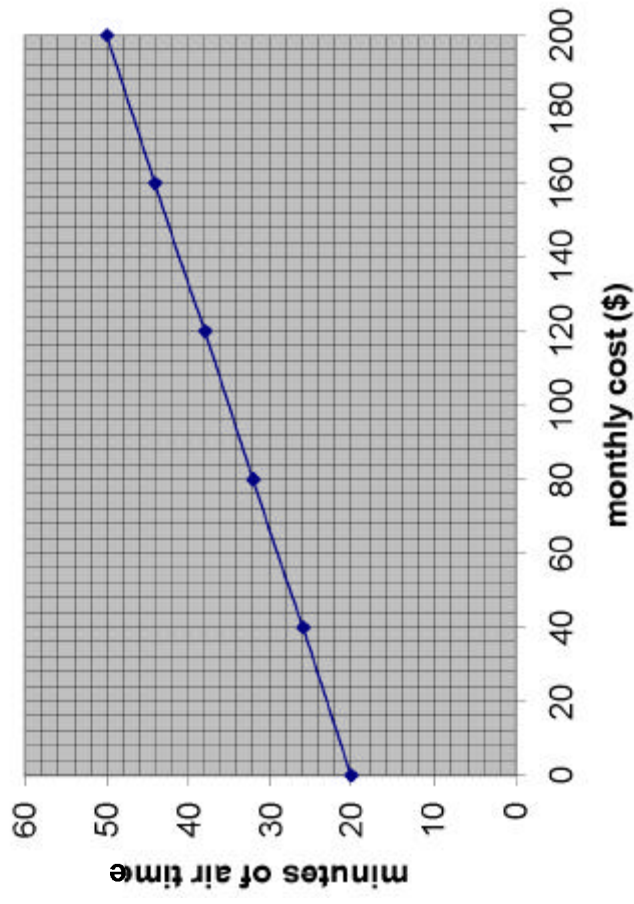
# CELL PHONES UNLIMITED



**CHECK OUR  
PRICES!**



OUR LOW COST PER MONTH





# A Phone Plan For You



Our cell phone phone plan is suited to your needs. Use the table below to check out our prices. We are certain that you will be satisfied! Call us at

1-800-999-9999

if you have any questions.

mintues	montly cost (\$)
0	\$9.95
10	\$12.45
20	\$14.95
30	\$17.45
40	\$19.95
50	\$22.45
60	\$24.95
70	\$27.45
80	\$29.95
90	\$32.45
100	\$34.95
110	\$37.45
120	\$39.95
130	\$42.45
140	\$44.95
150	\$47.45
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