PBS Mathline®

The Elementary School Math Project

Mirror, Mirror

Math Grows Up (Geometry/Spatial Sense)

Objective

Students will use spatial reasoning and problem solving strategies to determine which regular polygons will tessellate.

Overview of the Lesson

Students use hinged mirrors to discover that the regular polygons are composed of triangles tessellating around a center point. Students then sketch these triangles on paper models of the regular polygons having 3 to 10 sides and compute the measure of the center angles formed by these triangles in each of the different polygons.

Next, students are given opportunities to explore more geometric concepts as they use mirrors and paper polygons to determine which regular polygons tessellate. They are challenged to give reasons why some polygons tessellate and others do not.

Materials

Teacher:

- Large classroom chart (Copy of Activity Sheet: "Will It Tessellate?")
- Activity Sheet: "Polygon Cut-Outs" for template

Each Student:

- Copy of Activity Sheet: “On the Dot”
- Hinged Mirrors (formed by taping the edges of two small hand mirrors together)
- Copy of Activity Sheet: “Will It Tessellate?”
- Calculator

Each Student Pair:

- 10 cut-outs of each polygon (See Activity Sheet: “Polygon Cut-Outs” for template)

Procedure

Distribute a hinged mirror and a copy of Activity Sheet: “On the Dot,” to each student. Instruct students to place the vertex of their hinged mirror on the dot in the middle of the paper above the line. Have students describe to their partners what they see as they open and close the mirrors.

Next, have students identify the regular polygons that they see in the mirrors. Be certain that students understand that a regular polygon is a closed figure that has three or more equal sides and angles. ("Poly" means many and "gon" means angles.) Have students give the names for the regular polygons such as: Triangle (3 sides); Square (4 equal sides and angles); Pentagon (5 sides and angles); Hexagon (6 sides and angles); Heptagon (7 sides and angles); Octagon (8 sides and angles); Nonagon (9 sides and angles); Decagon (10 sides and angles), and make a list on the board.

*Note: When using the mirrors, all of the polygons formed are regular polygons. However, all polygons are not regular.

Instruct students to create any regular polygon with their mirrors and to look for any other shape that appears in the mirror when this polygon is formed. Have students share their findings. (Students see that all of the polygons are composed of triangles.)

Distribute one cut-out of each of the regular polygons (up to 10 sides) to each pair of students. Use Activity Sheet: "Polygon Cut-Outs" as the template for these polygons. Have students sketch the triangles they observed in the mirror onto each polygon cut-out. Encourage students to discuss, with their partners, ways that this task can be accomplished. After allowing sufficient time for exploration, have them share their discoveries with the entire class. (Putting a dot in the center of the paper and then drawing a straight line to each of the angles is one way of sketching the triangles.)

Next have students compute the measures of the angles formed around the point in the center. (The angles are congruent. Since one complete rotation around a point is 360 degrees, to find the measure of each of these angles, divide 360 degrees by the total number of angles.) To demonstrate, draw a square on the chalkboard. Sketch the four triangles. Each angle will be 90 degrees because 360 divided by 4 (total number of angles around the center point) is 90 degrees. Then challenge students to determine the measure of each of these angles in the remainder of their polygons and record these angle measurements on the polygon cut-outs. Allow students to use calculators to complete this task.

Have students once again place the mirrors on the dot. Then instruct them to place a vertex of a polygon cut-out on the dot and position the mirrors so that they frame two sides of this polygon. To keep students from becoming confused, instruct them to first flip the polygon over so that the measures of the angles written on the polygon will not be seen. Students look in the mirror to see if the polygon tessellates. (A polygon tessellates if there are no gaps or overlapping pieces when used to cover a surface.) Students record their findings on Activity Sheet: "Will It Tessellate?" and give reasons why each polygon does or does not tessellate.

Finally, distribute envelopes containing 10 cut-outs of each polygon to pairs of students. Have students use these shapes to verify the answers obtained when using the mirrors. After allowing time for exploration, examine and discuss each polygon and have students share their reasons why it will or will not tessellate. (A polygon will tessellate if the sum of its angles around a given point equals 360 degrees.) Display a large chart of Activity Sheet: "Will It Tessellate?" and record class findings as discussed.

Mathematically Speaking. . .

This lesson contains several key geometric points that students should discern from their explorations.

- All of the polygons created with the mirrors are regular polygons. By observing what happens when the mirrors are opened and closed, students can see the various polygons being formed and the number of triangles centered around the center point that form the polygon. The number of degrees in the measure of the angles around the center point can be determined by dividing 360 degrees by that number.
- The number of triangles shown is the same as the number of sides in the polygon and the number of angles in the polygon.

- The triangle, square and hexagon are the only polygons that will tessellate. This is true because the sum of the angles of these polygons, centered around a given point, is 360 degrees. The pentagon, heptagon, octagon, nonagon and decagon will not tessellate because the sum of the measures of their angles centered around a given point is greater than 360 degrees. Therefore, any regular polygonwith seven or more sides will not tessellate.

- This lesson only addressed regular polygons, however, all triangles, quadrilaterals and hexagons will tessellate.

Extensions & Connections

Have students make a tessellation using pattern blocks. Take a photograph of the tessellation and display it in the classroom.

Have students explore the possibility of combining two different shapes to make a tessellation.

Resources

Rotocenter Revolution! NCTM Student Math Notes • September 1995
Written by David Masunaga, Iolani School, Honolulu, HI 96826
National Council of Teachers of Mathematics, Reston, VA

Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

Standard 1: Worthwhile Mathematical Tasks

- It is sometimes difficult to know when to lead and when to allow a student to struggle with a difficulty. Can you give any insights or relay experiences you have concerning this issue?

Standard 2: Teacher's Role in Discourse

- The video teacher spoke about the ways she conveys her love of math to her students. How do you convey your love of math to your students? If you do not have a particular interest in math, how do you keep from conveying this disinterest to your students?

Standard 5: Learning Environment

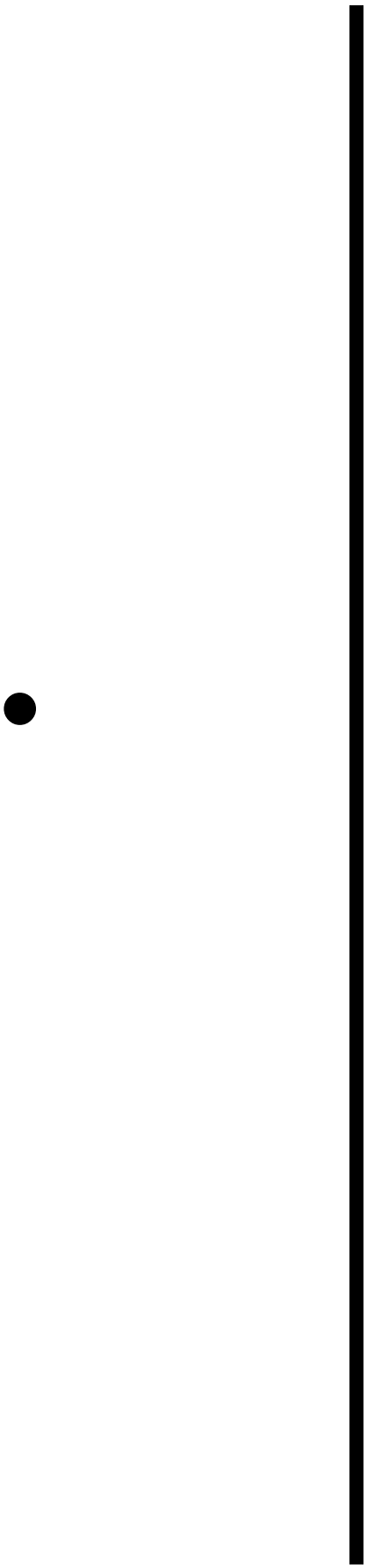
- A lesson of this type utilizes a lot of materials. Some of these materials have to be created, cut out, organized, distributed, and collected and maybe even purchased, i.e. mirrors. This can be quite a challenge. What are some of your best ideas on how you manage materials?... obtain materials?... create classroom sets of materials?

Standard 6: Analysis of Teaching and Learning

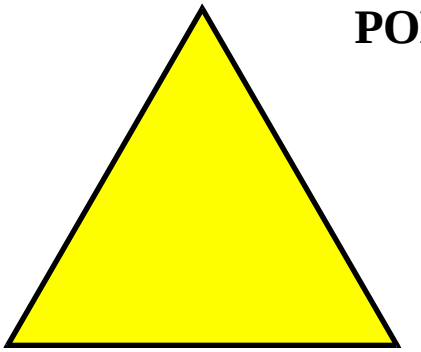
- How can you effectively assess individual student learning in a lesson where so much of the learning takes place in small groups?

- Providing so many opportunities for students to first share their thoughts and ideas with a partner or in a group offers many opportunities for teachers to hear what students are thinking. What role should the teacher assume when students are still in the process of discovery and synthesis?

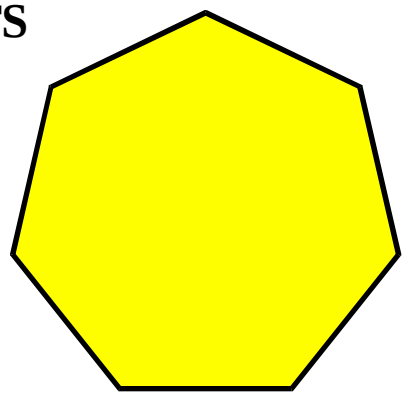
On The Dot



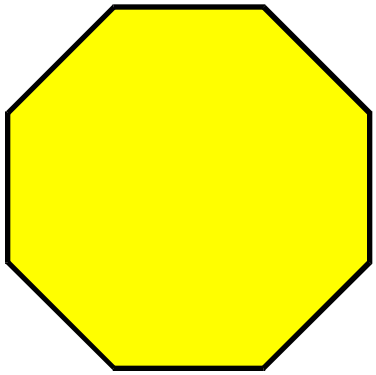
POLYGON CUT-OUTS



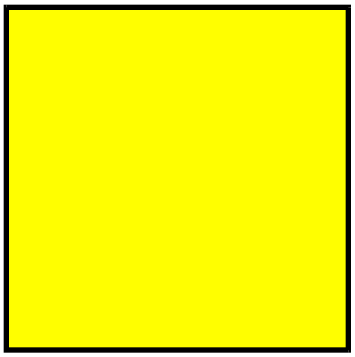
Triangle



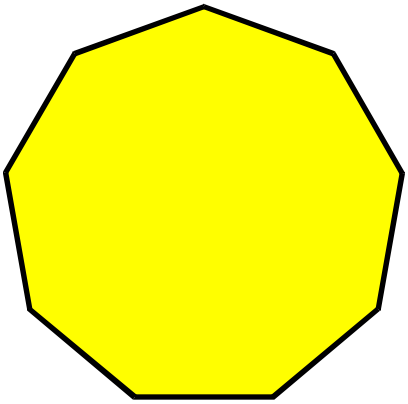
Septagon



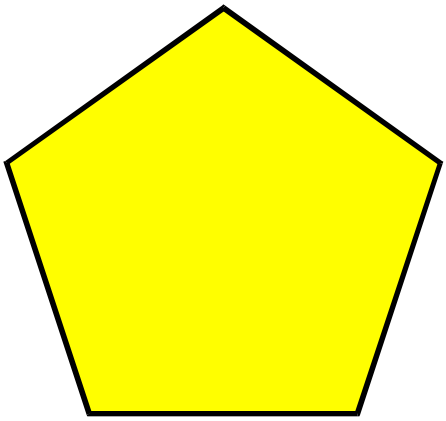
Octagon



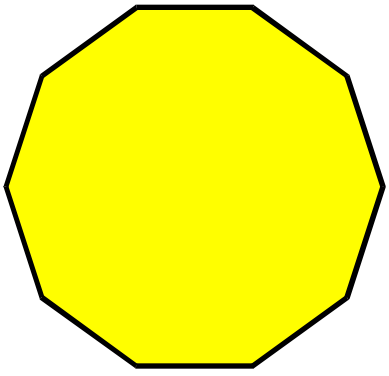
Square



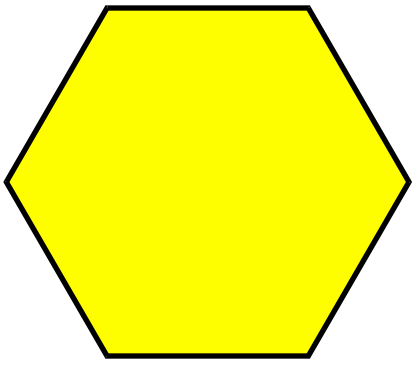
Nonagon



Pentagon



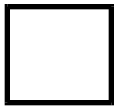
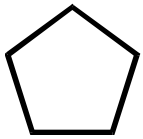
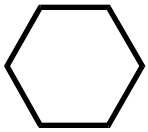
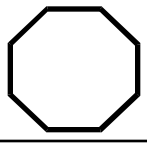
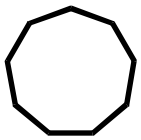
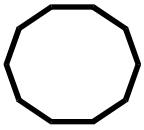
Decagon



Hexagon

Name_____
Date _____

Will it *Tessellate?*

Regular polygon shape	Name of regular polygon	Number of sides	Does it tessellate?	Why?
				
				
				
				
				
				
				
				

Your thinking...

Please answer in detail the following question. Use pictures and labels if necessary to explain your thinking.

Which shapes tessellate and why?