PBS Mathline®

The Elementary School Math Project

Sidewalk Capers

Math Grows Up (Geometry/Spatial Sense)

Objective

Students will explore spatial relationships by determining the area of various shapes and creating tessellation patterns.

Overview of the Lesson

Students use a variety of manipulatives in this lesson to explore spatial relationships of squares and their transformations. They create a square with 36 square tiles and determine that by sliding several of the pieces both vertically and horizontally, they form an irregular shape that can be tessellated. Students are given a variety of curved shapes and encouraged to use tape and scissors to reconfigure their shape back into a square. Students are surprised to learn that all of the different shapes have the same area. Given a variety of measurement tools, students are asked to estimate the number of squares tiles (24" x 24") needed to cover an assigned section of the school's sidewalk. Finally, they create a pattern from the square and tessellate it on the sidewalk using chalk.

Materials

Teacher:

- 2 squares: 36 square units in size. (One should show an outline of the grid)
- 36 magnetic square tiles or cut-outs of small squares that can be taped to the board
- 4 copies of the 36 square unit shape

Each Student:

- 36 square tiles

Each Student Pair:

- Copy of one of the Activity Sheets: "Special Shape" (A-E)
- Copy of Activity Sheet: "Sidewalk Capers"
- Colored sidewalk chalk
- Tape measure, tape, ruler, scissors
- Square unit made from paper (approximately 24" on a side)

Procedure

As a warm-up to the lesson, display two large paper squares on the board, each having an area of 36 square units. On the first square, draw a grid inside the square so that each square unit is displayed. On the second square, label each side of the square with its measurement of 6 units. Ask students to find the area of these two squares. (You may wish to remind students of the differences between area and perimeter.)

After allowing sufficient time, have students explain how they determined the area of the two squares. Some students may count the square units in each square, others may solve by multiplying, and others may use a formula. Prove to the students that the squares have the same area by placing one on top of the other. To extend student thinking, ask if the area of the square would change if the square is rotated? Demonstrate by rotating the shape on the board. Encourage students to give reasons for their answers.

Distribute 36 square tiles to each student and ask them to create a large square like the one on the board. Instruct them to take one tile from the right side of the square and move it straight across the same row to the other side. Now take another tile from the right side and move it straight across to the other side. Continue the same procedure to make several vertical slides by moving tiles from the top to the bottom. As you give students directions, demonstrate on the board with your 36 tiles. Tell the students that they no longer have a square but a new shape. Ask the students to determine the area of the new shape. Encourage students to give reasons for their answers. (The shape still has an area of 36 square units.)

Ask students if they can identify the term that refers to the situation where shapes fit together to cover the plane with no gaps or overlapping areas. (Tessellation). Inform the class that you have duplicated on the board the exact shape formed on

their desktops. Show the class four of these shapes and ask for volunteers to come to the board to show how they tessellate.

Divide the class into pairs and give each pair a copy of one of the Activity Sheets: "Special Shapes" (A, B, C, D, or E). Instruct students to estimate the area of their shape.

After allowing time for exploration and discourse, ask students to share their estimates and to discuss any obstacles they encountered in making these estimations. Demonstrate a technique for finding the area by transforming the original 36 tiles used earlier back into a square.

Challenge students to find the exact area of their shapes by using a similar method. Provide scissors and tape for them to use. Display all of the original shapes across the bottom of the board in alphabetical order. Have students tape their newly transformed shapes on the board above the original shape. If they compare the areas, they discover that all of the shapes have an area of 25 square units and that all can be reconfigured into a square. The concept that different shapes can have the same area should be emphasized.

Distribute a square unit to each pair of students (24" x 24"). Inform them that they will go outside where they will be assigned a section of the sidewalk. Their task will be to estimate how many of these square units will be needed to cover their section of the sidewalk. Once outside, provide students with measuring tapes, rulers, etc. to aid them in their estimations. Estimations and explanations should be recorded on Activity Sheet: "Sidewalk Capers."

Return to the classroom and discuss estimates with the class and why the estimations were not all the same. Finally, each student pair reconfigures their square to create a pattern piece to be used to tessellate their section of the sidewalk. Using tape and scissors, students use their creativity to design patterns. Once the pattern pieces are completed, return outside where students tessellate their section of the sidewalk by tracing and coloring their patterns.

Mathematically Speaking. . .

We often see fanciful repeating patterns in fabric designs, artwork, and tilings formed from polygon transformations. In this lesson, students explored ways to transform a square into a different shape with the same area that will still tessellate the plane. When students slide one square unit from one side of the square to the other they need to focus on the fact that the new protrusion on one side of the square can be matched with the indentation it left when it was removed from the other side. It is usually easier for the young learner to get a better mental picture of this with square pieces rather than curved ones. This is the reason students are

encouraged to cut the rounded shapes in this lesson and reassemble them back into the original square. When students make tessellating pieces for the sidewalk designs, they need to understand that the modified squares have the same area as the parent squares from which they were made and like the parent squares, they will also tessellate the plane.

Extensions & Connections

After the tessellations are complete, ask the students to state the number of shapes it took to cover their sidewalk section. Ask them to determine the cost of one tessellating piece if they were going to charge the principal of the school for their work. Have students then determine the cost for the entire cement block.

Have students examine the artwork of M. C. Escher. Challenge them to describe his tessellations and explore the polygon shapes he transformed to create his designs.

Resources

About Teaching Mathematics, Math Solutions Publications, 1992.

TesselMania!® DELUX: The Art, Design, and Gift Making Tool.
CD-ROM for Macintosh and Windows
MECC•6160 Summit Drive North•Minneapolis, MN 55430-4003
Tele: 612/569-1500•http://www.mecc.com/

The Graphic Work of M.C. Escher
Ballantine Books — New York (ISBN 0-345-27755-4
First Ballantine Books Edition: August 1971.

"Tessellations." *Mathematics Teacher* 66
L. Carey Bolster, April 1973, pgs. 339-42.

Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

Standard 1: Worthwhile Mathematical Tasks

- The video teacher states that one of the most challenging things is keeping everybody excited about learning math. Briefly describe a worthwhile task that was a real "turn on" for your students.

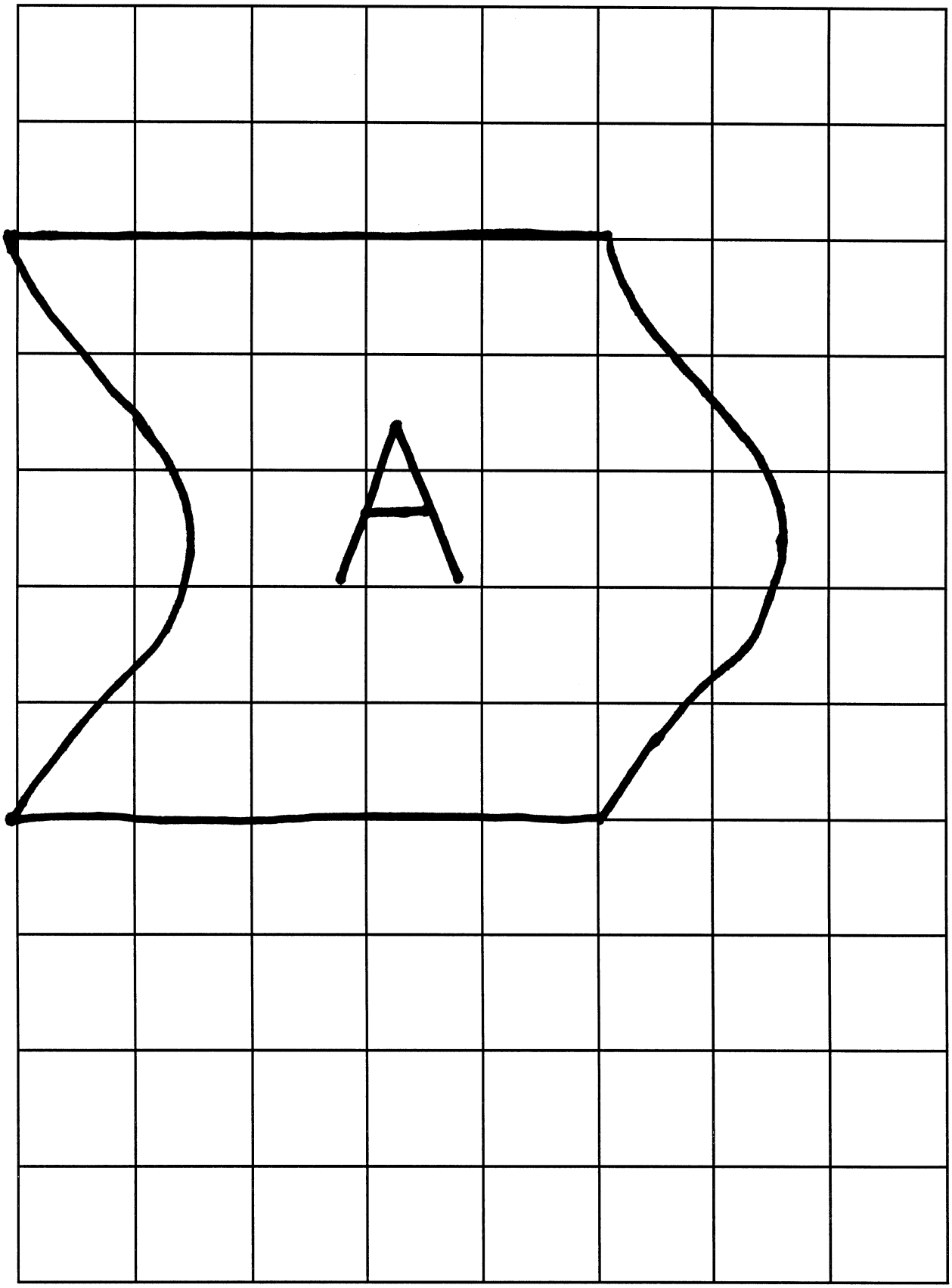
Standard 2: Teacher's Role in Discourse

- The video teacher draws students' names out of a can to determine who will answer his questions. What techniques have you used to make sure that all students contribute to classroom discussions?

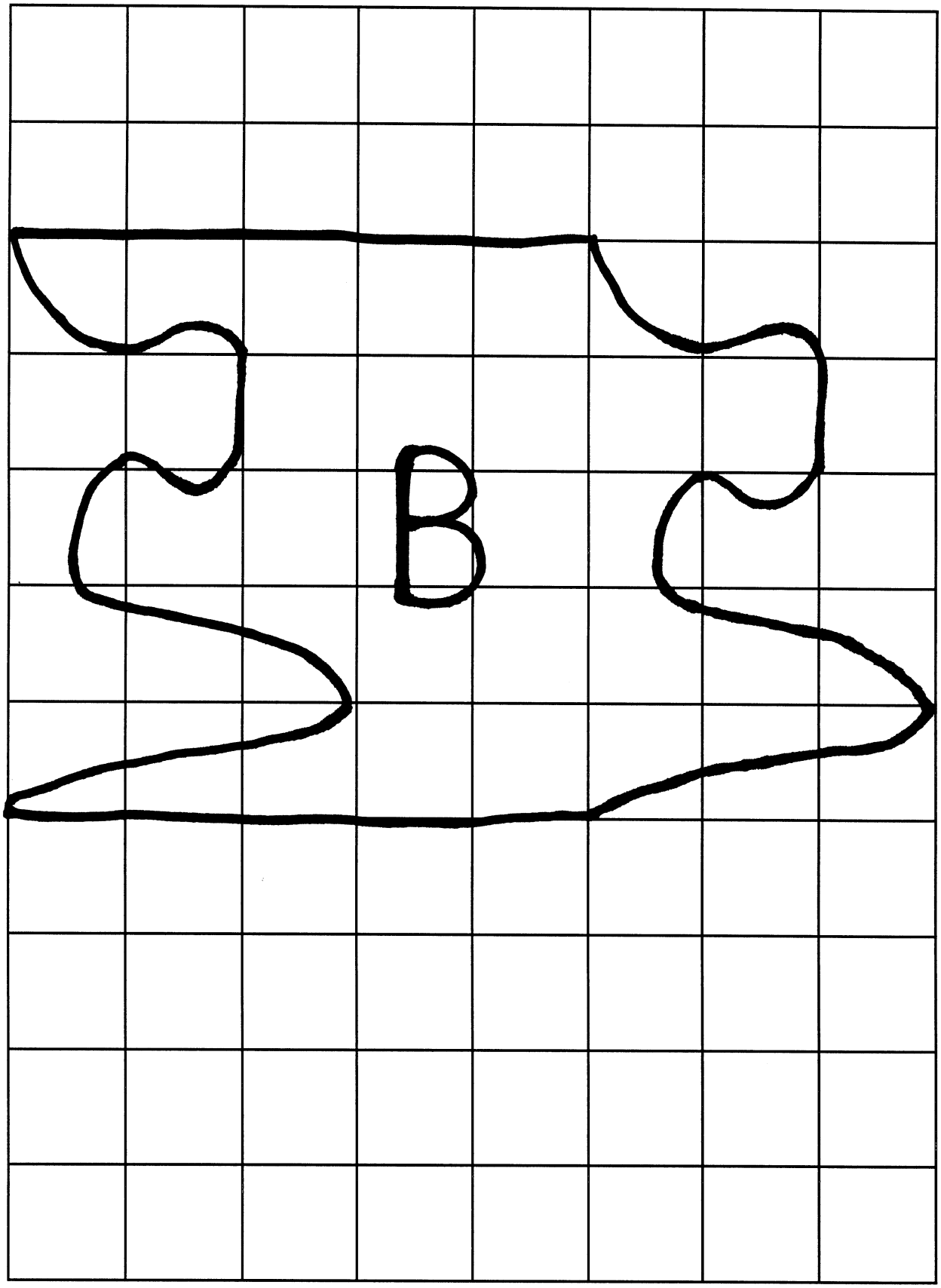
Standard 6: Analysis of Teaching and Learning

- The video teacher starts each lesson with a simple warm-up to launch the lesson. Share several of your favorite warm-up ideas.
- What are the benefits and the drawbacks to planning across grade levels? How is cross grade level planning and activities handled in your school?
- The key to the success of any lesson is careful planning and preparation. Share one of your secrets for effective and efficient planning and preparation.

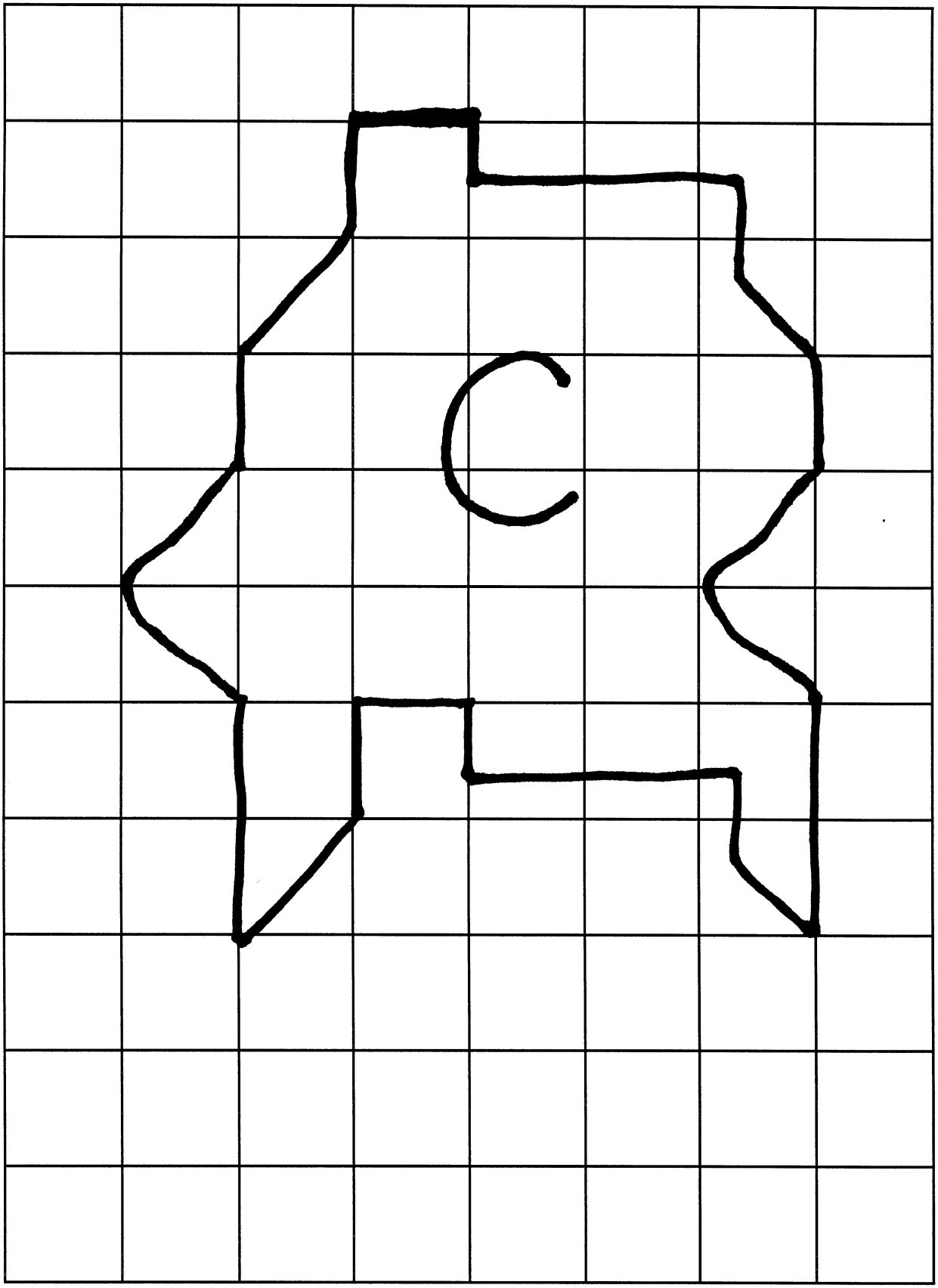
Special Shape A



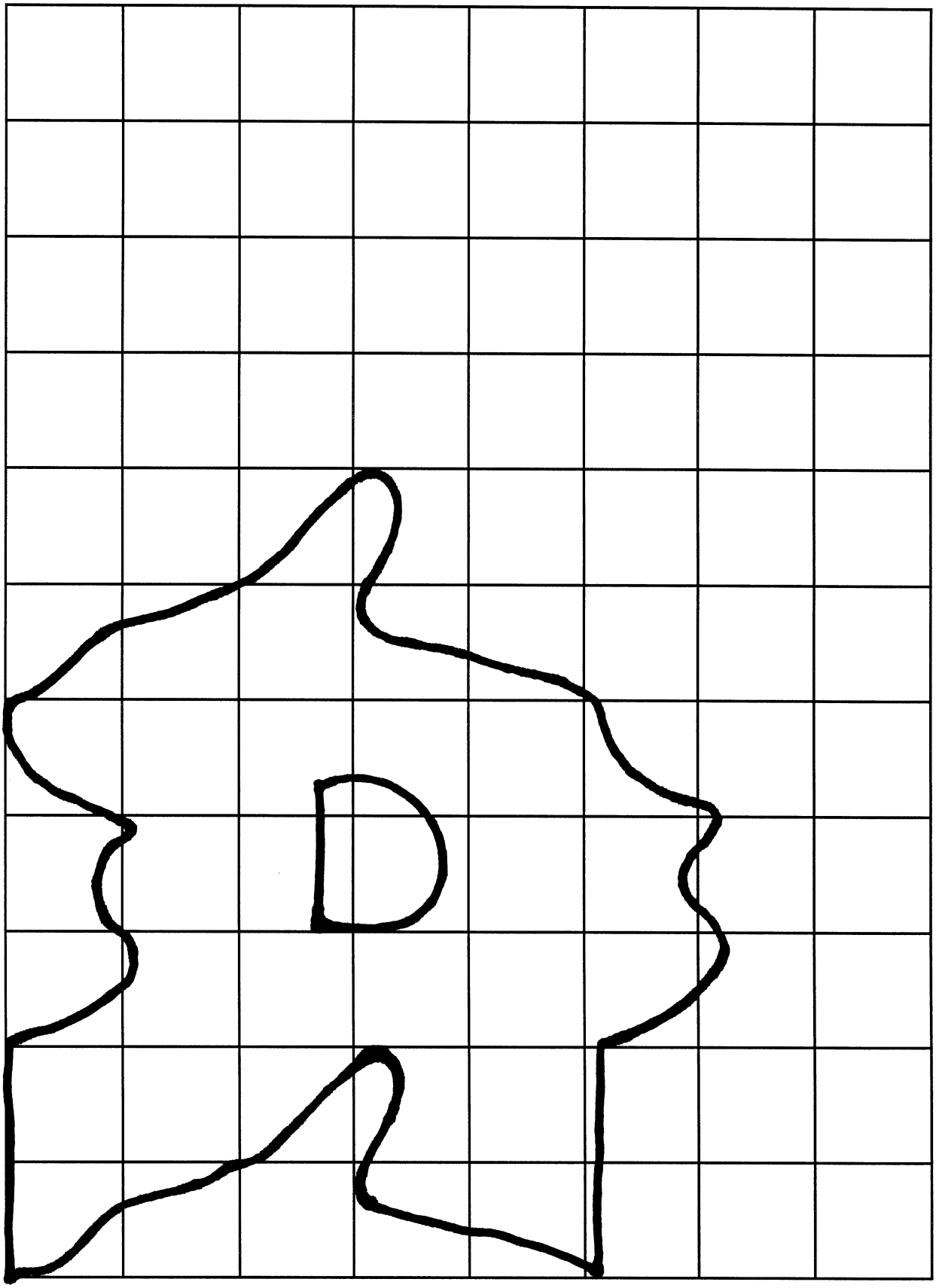
Special Shape B



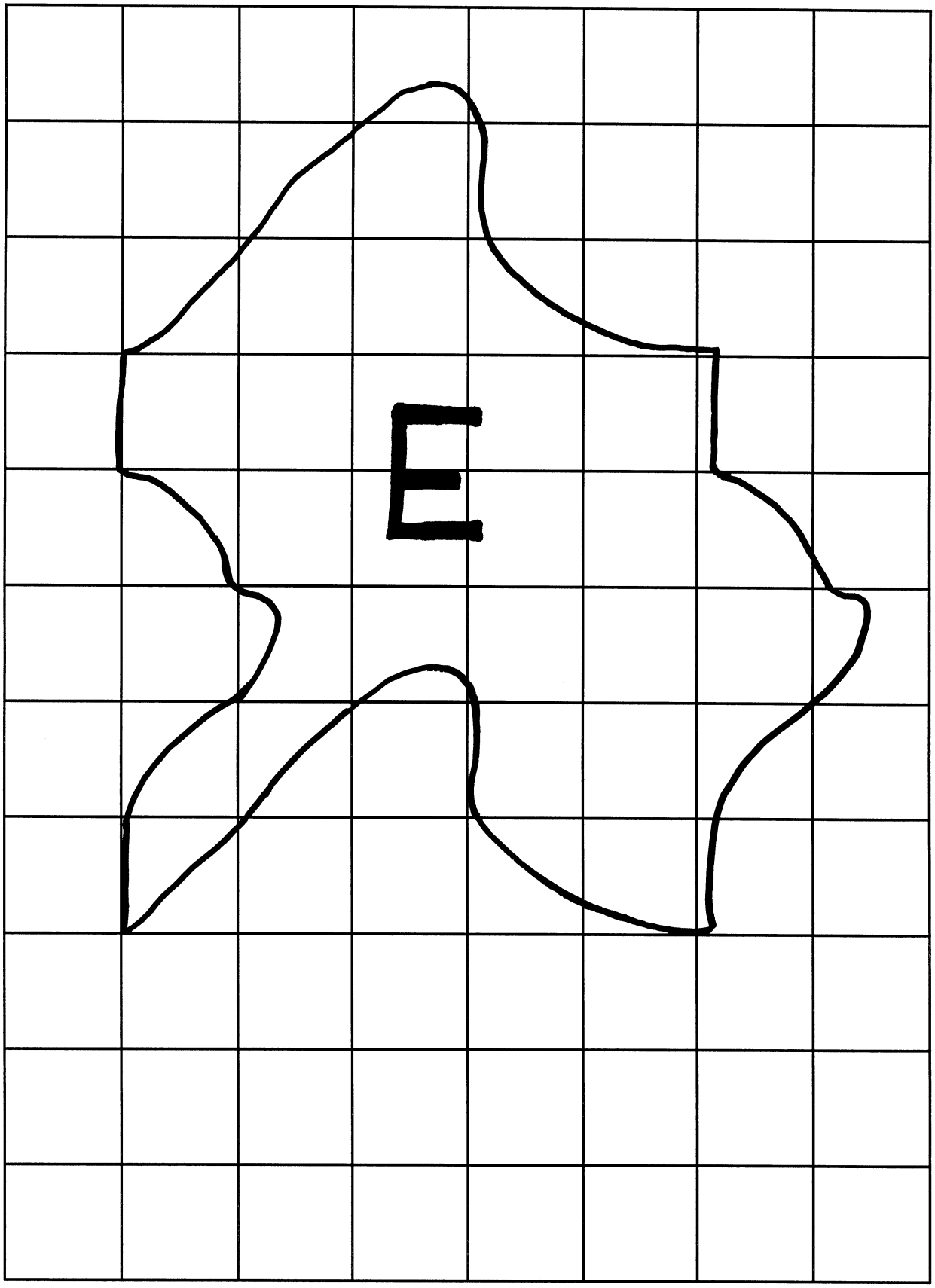
Special Shape C



Special Shape D



Special Shape E



Name _____

SIDEWALK CAPERS

About how many square tiles will it take to cover your concrete square?_____

How did you find your answer? _____

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_____

_____

— — — — — cut — — — — — — — — — cut — — — — — — — — — — — cut — — —

Name _____

SIDEWALK CAPERS

About how many square tiles will it take to cover your concrete square?_____

How did you find your answer? _____

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