The Busing Problem

(Linear Programming)

Objective

Students use substitution and the elimination method to solve linear systems in order to solve linear programming problems.

Overview of the Lesson

In this lesson, the students solve two linear programming problems. The first one. *Researching Research Papers*, involves three variables; the second, *The Busing Problem,* involves four variables. During the lesson, students work together to solve problems by exchanging ideas, organizing information, and sharing that information with the entire class. Students apply the substitution method and the elimination method to solving linear systems, which allows them to identify the corner points of the feasible regions. All of these corner points are tested in the equation that is to be maximized or minimized, depending on the particular problem situation. After solving the problems, students conclude that the problems with four variables are complex and very tedious to solve. This prepares them for using matrices and the graphing calculator to solve systems in *Meadows or Malls?* lesson that follows. Because of the sequential nature of the lessons, *The Busing Problem* should be done before *Meadows or Malls?*

Materials

- chart paper and markers
- overhead projector
- graphing calculators
- *Researching Research Papers* activity sheets
- *The Busing Problem* activity sheets

Procedure

1. **Researching Research Papers**: Have students read the problem for the class, and then have the groups discuss the problem and develop a list of constraints. After the groups have had time to work together, ask various groups to give a constraint. List those constraints on the board.

   In the Researching Research Problem, a student wants to maximize her grade. She figures she will earn 5 points for each page of original writing (W), 4 points for a page of quotations (Q), and just 1 point for each page of pictures (P). Based on the facts presented in the problem, help students determine the following constraints:

I.	$P + Q + W = 20$
II.	$P \geq 3$
III.	$0.5Q + W \leq 12$
IV.	$Q \geq 0$
V.	$W \geq 0$
VI.	$P \geq 0$

   Since the problem contains three variables, systems of three equations need to be considered. Because the first constraint is an equation, it must be included in all of the combinations tested. This means that the number of combinations will be ${}_5C_2$ or 10, the number of ways you can take five things two at a time. But, because any point that fits condition II will also fit condition VI, this constraint does not need to be used. Now, only 6 combinations, ${}_4C_2$, need to be checked: (I, II, III), (I, II, IV), (I, II, V), (I, III, IV), (I, III, V), and (I, IV, V).

2. **Solving the Systems for Researching Research Papers**: After the groups all report and put the list of constraints on the board, have students work within their groups to determine the solution to the problem. They solve three variables and three equations at a time, using whatever method they want to use. After each group finishes their calculations and agrees upon the answer, have the students in the group record their work and their solution on chart paper.

3. **Presentation of the Solutions to the Systems**: Have students present their solutions to the problem using chart paper to display their work and answers. After all student groups are done, make sure that three different methods of determining the solution get presented: listing a table of values, substitution, and the elimination method.

   The solution of a linear programming problem is found at one of the corners of the feasible region. Using the equation that gives the grade as a function of the numbers of pages of pictures, quotations, and original writing, test the vertices of the corner points to determine which point gives the maximum grade. This is the final step in determining the solution. In all groups, regardless of the method used, students should have determined that the maximum grade is 78.

Combination	P	Q	W	Grade
I, II, III	3	10	7	78
I, II, IV	3	0	17	Violates III
I, II, V	33	17	0	71
I, III, IV	8	0	12	68
I, III, V	–7	24	0	Violates II
I, IV, V	20	0	0	20

4. **The Busing Problem**: After completing the *Researching Research Papers Problem*, students move on to *The Busing Problem*. Have students read the problem aloud. This problem deals with four variables:
   E_e is the number of East High student living on the east side of the river.
   W_e is the number of West High students living on the east side of the river.
   E_w is the number of East High students living on the west side of the river.
   W_w is the number of West High students living on the west side of the river.

   Have students work in groups to list the constraints, and then help them work as a class to list the constraints on the board. Direct students to make sure that they have the constraints numbered the same way on their paper as they are numbered on the board.

   I. $E_e + W_e = 300$
   II. $E_w + W_w = 25$
   III. $E_e + W_w \leq 350$
   IV. $W_e + W_w \leq 225$
   V. $E_w \geq E_e$
   VI. $E_e \geq 0$
   VII. $W_e \geq 0$
   VIII. $E_w \geq 0$
   IX. $W_w \geq 0$

5. **Listing the Combinations to be Solved**: Once the constraints are listed, students should work in groups to list all of the possible combinations. In this case, the first 2 constraints must be part of every group of 4, which leaves ${}_7C_2 = 21$ systems to consider. Ask for student volunteers on chart paper so the entire class can see them, the teacher divides the systems among the groups. Each group solves their systems and posts the results on the class chart.

6. **Homework is Assigned**: You may find that some of the answers posted need to be checked again. For homework, have the students check their work and come prepared to finalize the problem the following day.

Assessment

The video lesson presented many opportunities for the teacher to assess student learning. In both the group work and the large group presentations, the teacher listened to the thinking reflected in student statements and questions. From this process of listening, reflecting, questioning, and listening again, the teacher determines the guidance needed by the students.

Students share responsibility for the assessment process. Through the process of the small group work followed by large group presentations, the students in this class assess their own thinking and the thinking of their classmates. Helping students learn to analyze, formulate problems, and communicate mathematical ideas is all part of helping students become better able to reflect thoughtfully on and assess their own mathematical growth.

One student in the video mentioned seeing growth when he looked at his portfolio. He said that he learned new techniques and new methods and was able to really see that the processes are getting more complicated and his mind is expanding. Helping students monitor and guide their own development is one very good reason for using portfolios and projects in the mathematics classroom. Assessment of this sort—that is connected to instruction, that involves the students, and that helps the student analyze growth—has not been part of many traditional mathematics classrooms, but teachers, students, and parents are beginning to see the benefits of including it.

Extensions & Adaptations

- Why does the elimination method for solving systems work? Have your students complete the extra activity included in this Lesson Guide in order to help them investigate this question. This particular activity is appropriate for students just learning to solve systems as well as those students who already know how to solve systems.

- Ask students to prove algebraically why the elimination method works. This proof is presented in *Mathematically Speaking*.

- The activities in this lesson are complex, and for that reason an easier problem is included in this lesson guide, *Programming a Profit*, for classes needing an introductory level activity.

Mathematically Speaking

In many textbooks, students learn to solve systems of equations using three different methods: graphing, substitution, and elimination (sometimes called linear combination). The procedures are demonstrated, examples are given, and students are generally fairly successful with the algebraic manipulations. But do students really understand what they are doing? For the graphical and substitution methods, most students can easily understand why the methods work, but perhaps they might benefit from a closer look at the elimination or linear combination method.

The activity entitled *Illuminating Elimination* is designed to allow students to discover what is happening from a graphical point of view. This may give more meaning to the algebraic manipulations that they perform. In the activity, the students start with a system of two linear equations that they graph to determine the solution. Then they add the original equations and graph the results. They continue to take various combinations of the original equations and graph the results. In every case, the new line passes through the original intersection point. The students should come to understand that if they can find the linear combination that gives the vertical and horizontal lines, they can determine the x and the y coordinates of the intersection point. Teaching for understanding, and helping students know why particular methods work is an important part of making mathematics meaningful.

You might ask your students to show algebraically why the elimination method works. Given a system of two linear equations:

$$ax + by = e$$
$$cx + dy = f, \text{ where } bc - ad \neq 0$$

show that their intersection point is also a point on the line that is formed by adding or subtracting these equations after either or both equations are multiplied by a constant.

Solving the linear systems using matrices gives:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1}\begin{bmatrix} e \\ f \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{d}{ad-bc} & \frac{-b}{ad-bc} \\ \frac{-c}{ad-bc} & \frac{a}{ad-bc} \end{bmatrix}^{-1}\begin{bmatrix} e \\ f \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{de}{ad-bc} - \frac{bf}{ad-bc}, & \frac{-ce}{ad-bc} - \frac{af}{ad-bc} \end{bmatrix}$$

$$(x,\ y) = \left(\frac{de-bf}{ad-bc}, \frac{af-ce}{ad-bc}\right).$$

Now we show that if either or both of the original equations are multiplied by a constant and the resulting equations are combined by addition or subtraction, the new equation also passes through the intersection of the original equations, $\left(\frac{de-bf}{ad-bc}, \frac{af-ce}{ad-bc}\right)$.

$$\begin{aligned} &(1) \quad kax + kby = ke \\ &(2) \quad hcx + hdy = hf, \text{ where } bc - ad \neq 0,\ k \neq 0,\ h \neq 0 \end{aligned}$$

Adding (1) and (2) gives: $(ka + hc)x + (kb + hd)y = ke + hf$

In order to show that this new line also passes through $\left(\frac{de-bf}{ad-bc}, \frac{af-ce}{ad-bc}\right)$, we need to show that substituting this point into the previous equation gives a true statement.

$$\begin{aligned}
(ka + hc)\left(\frac{de-bf}{ad-bc}\right) + (kb - hd)\left(\frac{af-ce}{ad-bc}\right) &= ke + hf \\
\frac{kade - kabf + hcde - hbcf}{ad-bc} + \frac{kabf - kbce + hadf - hcde}{ad-bc} &= ke + hf \\
\frac{kade - hbcf - kbce + hadf}{ad-bc} &= ke + hf \\
\frac{(adke + adhf) - (bchf + bcke)}{ad-bc} &= ke + hf \\
\frac{ad(ke + hf) - bc(ke + hf)}{ad-bc} &= ke + hf \\
\frac{(ad - bc)(ke + hf)}{ad-bc} &= ke + hf \\
ke + hf &= ke + hf .
\end{aligned}$$

Thus, we have shown that any combination passes through the intersection point of the original lines.

Hopefully, the students will understand that the most convenient combinations will be when y is eliminated so we know the x-coordinate of the intersection point, and when x is eliminated which will give the y-coordinate of the intersection point.

Tips From Ellen

Keep the Momentum: Implementation Tips

Change is a difficult process, and sometimes we get so caught up in the day-to-day events that our good intentions are lost. The following ideas are excerpted and adapted from a booklet on Interactive Mathematics Program (IMP) Implementation. Some ideas are probably obvious by now, so consider this a checklist and give yourself credit for all you've done, rededicate yourself to areas you've let slide, and pick at least one new idea to try!

Read/Reread the Standards Documents
In addition to reminding us what we are about in mathematics reform, they also have some great ideas!

Visit Other Sites
Visit other classrooms and schools where the standards are being implemented. Sit with students as they work and listen to their thinking. Steal ideas!

Visit Feeder Schools
Meet with the mathematics departments to talk about secondary mathematics reform and how your programs can connect and support each other. Have each department create a "wish list" for the other to get the dialogue going.

Develop Support Within Your Own School
Invite your colleagues to visit your classroom. Ask for feedback on one or two techniques you are exploring. Work to create an atmosphere of trust and support. (Maybe they'll reciprocate!)

Question the Textbook
Question why you are teaching what you are teaching, the order in which you teach it, and the emphasis you place on different topics. Consider how other textbooks approach the subject. Engage your colleagues in these discussions. See the NCTM *Standards* for guidelines and suggestions.

Talk with Administrators
Talk about the direction in which the mathematics department would like to go; discuss ideas; and outline the support you will need.

Parent Night
Share the direction that mathematics education is taking and have parents participate in some mathematics activities. Reassure them if needed. For example, inform them that graphing calculators have been approved by the College Board for use on AP Calculus tests!

Attend Conferences
Attend mathematics conferences at the local, state, and national level. Get new ideas, and form new networks.

Learn Something New
Consider learning not only new ways to approach mathematics topics that you have studied in the past, but also some mathematics topics that may be new to you. Learning along with your students is a powerful experience!

Resources

Foerster, Paul, *Algebra and Trigonometry, Functions and Applications*. Menlo Park, CA: Addison-Wesley Publishing Company, 1990. This text has an excellent section linear programming.

Introduction and Implementation Strategies for the Interactive Mathematics Program, A Guide for Teacher-Leaders and Administrators. Berkeley, CA: Key Curriculum Press, 1997.

Key Curriculum Press, PO Box 2304, Berkeley, CA 94702, 510-548-2304, sales@keypress.com, http://www.keypress.com

National Council of Teachers of Mathematics. *Assessment Standards for School Mathematics*. Reston, Virginia: National Council of Teachers of Mathematics, 1995.

Internet Location: **http://www.mcs.anl.gov/home/otc/Guide/faq/linear-programming-faq.html.** This is a rich source of information about linear programming. It includes a list of frequently asked questions along with answers such "What is Linear Programming?", "Where is there good software to solve LP problems?", "What is a modeling language?", "What software is there for Network models?", "What software is there for the Traveling Salesman Problem (TSP)?", and "What software is there for the Knapsack Problem?"

Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

Standard 1: Worthwhile Mathematical Tasks

1. Linear programming is one of the most widely used applications of mathematics with uses in business planning, industrial engineering, and social and physical sciences. What do students gain from the large unit problem approach to this topic?

2. At one point in the video, a student comments on the fact that in this course there is an emphasis on the meaning of the algebra, and not simply working with symbols that do not make any sense. How important do you think this is to students? What do you do to show that algebra is meaningful?

Standard 2: The Teacher's Role in Discourse

3. The video teacher says at one point, "They would love to have me tell them if it's right or wrong, but I know that in the end it's best for them to struggle with it, have some confusion, and with some assistance and facilitation come upon the solution and really take it as an achievement on their own." What can teachers do assist and facilitate rather than just give solutions?

Standard 3: Student's Role in Discourse

4. From the comments of both the teacher and the students, the viewer knows that there is an emphasis on teamwork in this class. The students work cooperatively, share ideas, and solutions, and check each other's work. How do you develop this attitude of respect and cooperation in a mathematics class? How do you assist students in understanding the importance of this type of group work?

Standard 4: Tools for Enhancing Discourse

5. At several times during the lesson, students record their findings, ideas, and solutions on newsprint, and various groups make reports to the entire class. Why is this important for the teacher? Why is this important for the students?

6. At one point in the lesson, the teacher brought out a 3-D model that students had created in a previous unit. In what ways was the old 3-D model helpful?

Standard 5: Learning Environment

7. Part of creating a positive learning environment means that the teacher consistently expects and encourages students to display a sense of mathematical competence by validating and supporting ideas with mathematical argument. How does the video teacher accomplish this?

Standard 6: Analysis of Teaching and Learning

8. In this lesson, the teacher comments, "It's very important for the students to be able to verbalize their thinking, and the process that they're using to solve the problem on their own. And if they can verbalize it on their own . . . research has shown that they will be more likely to retain the material." How do you use oral presentations in your classroom? What part of the student grade should be based on these types of presentations?

Researching Research Papers

Clarabell, who thought she had found the way to an 'A' on IMP unit assessments in *Is There Really a Difference?*, has turned her attention to writing research papers. She happens to have a research paper due soon, which is to be exactly 20 pages.

Most students in her school use a combination of their own original writing and quotations from other sources in their research papers. But Clarabell has heard that "a picture is worth 1,000 words." (Pasting in pictures also happens to be easier than writing.) Clarabell has lots of pictures, including three full-page pictures that she really likes. She has decided that those three pictures definitely have to be part of her 20 page paper. She just isn't sure whether or not to use other pictures as well.

She is a bit pressed for time, with her very active social life, and figures she can only spend 12 hours writing and collecting quotations. She figures it takes about an hour to do a page of original writing, and half an hour to collect a page of good quotations.

Now, Clarabell realizes that her teacher may not be happy with lots of pictures in a research paper. Since the paper is to be 20 pages long, she has decided that her teacher will be giving a maximum of 5 points per page. She figures that she will get the full 5 points for her pages of original writing, but that a page of quotations is only worth 4 points, and that a page of pictures is only worth 1 point.

So, based on her expectations of the grading system, how many pages of each type of material—original writing, quotations, and pictures—should she use to maximize her grade? What's the highest grade she can get? Explain your method.

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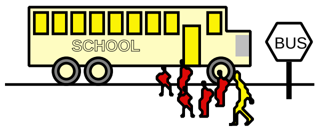
The Busing Problem



There are only two high schools in River City. People in River City aren't very creative with their names. East High is on the east side of the river, and West High is on the west side.

In the past, the students who lived on the east side of the river went to East High, and those living on the west side went to West High. Since the city was spread out, some students needed to be bused to their schools.

Two things have led to a need to change the way students are assigned to schools:

- West High School has become overcrowded, while East has had extra room.
- Recently, a federal court ordered River City to integrate the two schools. In particular, they mandated that at least half of the students at East High school should come from the west side of the river.

Here are some facts about the situation:

- There are 300 high school students living on the east side of the river and 250 living on the west side.
- East High School can handle up to 350 students, and West High School can handle up to 225.
- The average cost for busing per day will be:
  - ❒ $1.20 for each east side student going to East High;
  - ❒ $2.00 for each east side student going to West High;
  - ❒ $3.00 for each west side student going to East High; and
  - ❒ $1.50 for each west side student going to West High.

The problem facing the River City school board (and you) is to find out how many students to send to each school so that the busing costs are minimized.

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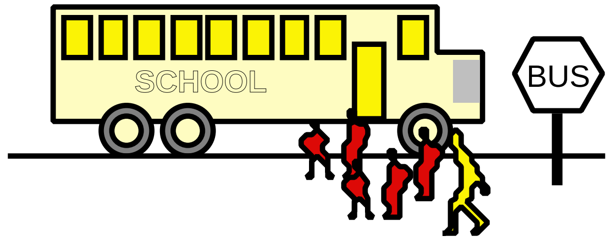

Busing—Part I

1. Write out the constraints—i.e., the equations and inequalities—that describe the problem. If everyone uses the same variables, it will make discussion of the problem easier. Please use the following:

   - E_e is the number of East High students who live on the east side of the river;
   - W_e is the number of West High students who live on the east side of the river;
   - E_w is the number of East High students who live on the west side of the river;
   - W_w is the number of West High students who live on the west side of the river.

2. Decide what combinations of constraints you will need to examine.

Busing—Part II

1. Solve the various combinations of equations that you think you must look at.

2. Based on your solutions, write up your recommendation for the school board.

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Illuminating Elimination

Solving Linear Systems

One might go about solving a linear system in several ways. In this activity, you investigate the elimination method (sometimes called the linear combination method) as you follow the steps below. After completing these questions, you should be able to describe the elimination method, and and explain why it works.

1. Solving a system means determining all points that the equations have in common. One easy way to do this is to graph both equations in the same coordinate plane. Graph the following system and determine the solution.

$$x + y = 8$$
$$x - 2y = -10$$

2. Now add these linear equations together to get a third linear equation. Graph this line in the same coordinate plane. What do you notice?

3. Multiply the first equation by 3, and add it to the second equation. Again, graph this new line in the same coordinate plane, and record your observations.

4. What do you think will happen if you multiply either one or both equations by constants other than one and then add the new equations? Test your conjecture by trying several combinations. Describe what happens.

5. When you are solving a system, some combinations are more helpful than others. Multiply the first equation by –1 and add it to the second equation. Graph this new equation. What do you notice about the graph? What do you notice about the symbolic form of the new equation? What information do you now have that is helpful in solving the system?

6. Use the strategy demonstrated in question 5 to eliminate the y terms and solve for x. Be sure to graph the resulting equation. What is the solution to the system?

7. Describe the elimination method, and explain why it works.

Programming a Profit

The Cheerleaders at Atholton High School needed money for new uniforms. They decided to sell T-shirts and hats to raise some funds. The principal told the cheerleaders that she would loan them up to $2,200 for the shirts and hats if they could make a profit and pay back the loan within two months.

In order to make the best decisions about how many hats and shirts to buy, the cheerleaders wanted to plan carefully. They knew the shirts would cost them $8 each, and the hats would run $6 each. They planned to sell the shirts for $11 and the hats for $10. They also knew that they could sell at least twice as many shirts as hats, and that the company from which they were ordering had a minimum requirement of 50 units of each item. In order to make the most profit, how many shirts and hats should they order?

1. What variables are associated with this problem?

2. List the restrictions stated in the problem.

3. Write a profit statement.

4. Graph the feasible region for the solution set.

5. Solve the appropriate systems in order to identify the vertices of the feasible region.

6. Test the vertices of the feasible region in your profit equation. How many shirts and hats should be ordered for a maximum profit to be made, assuming all items will be sold?

Researching Research Papers

Selected Answers

Based on the facts presented in the problem we have the following constraints:

I.	$P + Q + W = 20$
II.	$P \geq 3$
III.	$0.5Q + W \leq 12$
IV.	$Q \geq 0$
V.	$W \geq 0$
VI.	$P \geq 0$

Since the problem contains three variables, systems of three equations need to be taken. Because the first constraint is an equation, it must be included in all of the combinations tested. This means that the number of combinations is ${}_5C_2$. But, because any point that fits condition II will also fit condition VI, constraint VI does not need to be used. Thus, only 6 combinations, ${}_4C_2$, need to be checked: (I, II, III), (I, II, IV), (I, II, V), (I, III, IV), (I, III, V), and (I, IV, V).

Based on the equation, Grade = 5W + 4Q + P and the solutions for the following combinations, we have the possible grades. The maximum grade that can be earned is 78: when 3 pictures, 10 pages of quotations, and 7 pages of original writing are used.

Combination	P	Q	W	Grade
I, II, III	3	10	7	78
I, II, IV	3	0	17	Violates III
I, II, V	33	17	0	71
I, III, IV	8	0	12	68
I, III, V	–7	24	0	Violates II
I, IV, V	20	0	0	20

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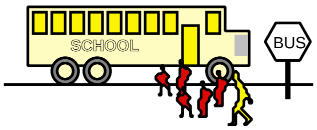
The Busing Problem



Selected Answers

Part I.

1.

I.	$E_e + W_e = 300$
II.	$E_w + W_w = 25$
III.	$E_e + W_w \leq 350$
IV.	$W_e + W_w \leq 225$
V.	$E_w \geq E_e$
VI.	$E_e \geq 0$
VII.	$W_e \geq 0$
VIII.	$E_w \geq 0$
IX.	$W_w \geq 0$

2. Students should know that they need to examine the corresponding linear equations in sets of four. All sets must include I and II because they are equations. Each set of four must have I and II and two additional equations that correspond to two of the other seven inequalities. This means students must consider ${}_7C_2 = 21$ systems.

Part II.

1.

Combination	E_e	W_e	E_w	W_w	Constraint(s) Violated or Costs
I, II, III, IV	Inconsistent				
I, II, III, V	175	125	175	75	$1097.50
I, II, III, VI	0	300	350	–100	Violates IX
I, II, III, VII	300	0	50	200	Violates V
I, II, III, VIII	350	–50	0	250	Violates V, VII
I, II, III, IX	100	200	250	0	$1270
I, II, IV, V	162.5	137.5	162.5	87.5	$1088.75
I, II, IV, VI	0	300	–75	325	Violates IV, V, VIII
I, II, IV, VII	300	0	25	225	Violates V
I, II, IV, VIII	325	–25	0	250	Violates V, VII
I, II, IV, IX	75	225	250	0	$1290
I, II, V, VI	0	300	0	250	Violates IV
I, II, V, VII	300	0	300	–50	Violates III, IX
I, II, V, VIII	0	300	0	250	Violates IV
I, II, V, IX	250	50	250	0	Violates III
I, II, VI, VIII	Inconsistent				
I, II, VI, VIII	0	300	0	250	Violates IV
I, II, VI, IX	0	300	250	0	Violates IV
I, II, VII, VIII	300	0	0	250	Violates IV, V
I, II, VII, IX	300	0	250	0	Violates III, V
I, II, VIII, IX	Inconsistent				

2. Thus, the feasible region for this problem has only four corner points. The apparent best solution to the problem is: $E_e = 162.5$, $W_e = 137.5$, $E_w = 162.5$, and $W_w = 87.5$, which gives a cost of $1,088.75.

   But, of course, you can't send half a student to one school and the other half of that student to another, so this isn't really the answer. It makes sense that we should round off the values one way or another. But it is not at all obvious how to do this rounding.

   Because of constraints I and II, if E_e goes up, then W_e goes down, and vice versa, and similarly for E_w and W_w. So students must consider four cases. Of the four cases, the only ones that fit all the constraints are the following two:

$$E_e = 162,\ W_e = 138,\ E_w = 163,\ W_w = 87, \text{ and}$$

$$E_e = 163,\ W_e = 137,\ E_w = 163,\ W_w = 87.$$

The first gives a cost of \$1089.90, while the second gives a cost of \$1089.10, so the latter is the least expensive arrangement.

(Having $E_e = 162$, $W_e = 138$, $E_w = 162$, $W_w = 88$ violates constraint IV; and having $E_e = 163$, $W_e = 137$, $E_w = 162$, $W_w = 88$ violates constraint V.)

You should point out that the difference in cost between the two cases that fit the constraints is pretty trivial. The apparent best whole number solution costs \$1089.10, and the best of all solutions costs \$1,088.75, so the whole number solution we have found would at worst be costing an extra 35 cents.

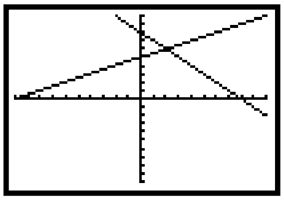
In real life, the cost of searching for a possible improvement would be far greater than the savings if one were found.

Illuminating Elimination

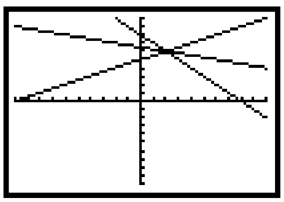
Solving Linear System

Selected Answers

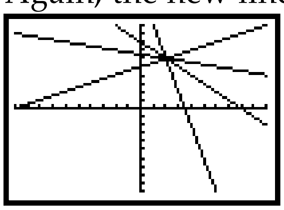
1. The lines cross at the point (2, 6).



2. The new line crosses at the same point.



3. Again, the new line crosses at the same point.



4. Students should see from testing other points that any combination of the two lines will give a line that also passes through the same point.

5.
$$\begin{array}{r} -x - y = -8 \\ x - 2y = -10 \\ \hline -3y = -18 \\ y = 6 \end{array}$$

By eliminating the x-terms you can solve for the y-coordinate of the intersection point.

6. By multiplying the first equation by 2 and adding it to the second, you can solve to determine that $x = 2$.

7. See *Mathematically Speaking*.

Programming a Profit

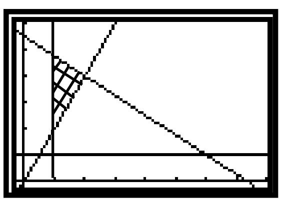
Selected Answers

1. T = number of T-shirts
   H = number of hats

2. I. $8T + 6H \leq 2200$
   II. $H \geq 50$
   III. $T \geq 50$
   IV. $T \geq 2H$

3. Profit = $3T + 4H$

4.



5.

Combinations	Hats	T-shirts	Profit
I, II	50	237.5	$912.50
I, III	300	50	Violates IV
I, IV	100	200	$1000
II, III	50	50	Violates IV
II, IV	50	100	$500
III, IV	25	50	Violates II

6. Maximum profit will be earned by ordering and selling 100 hats and 200 T-shirts.