PBS Mathline®

*The High School Math Project —Focus on Algebra*

# Mystery Liquids

(Linear Functions)

## Objective

Students will collect mass and volume data for two mystery liquids, create scatterplots and graphs, calculate linear models, and interpret their results.

## Overview of the Lesson

The Mystery Liquids lesson is a data analysis lab in which students gather mass and volume data for two mystery liquids, oil and water, and then use the data to explore linear functions. Connections to science are an important part of this algebra lesson, which deals with density. Active learning and communication are aspects of the lesson that are modeled for teachers. Students discuss the physical meaning of the slopes and y-intercepts of the various lines they have created from their scatterplots.

## Materials

- fish tank (or other large, clear container) full of water
- 1 can of Diet Coke
- 1 can of regular Coke

For each group of four:

- balance
- two graduated cylinders
- liquid A (water with red food coloring added)
- liquid B (oil)
- large-size graph paper
- red and blue marking pens
- meter sticks
- graphing calculator
- *Mystery Liquids* activity sheets

## Procedure

1. **Introductory Activity**: Ask the students what they think will happen when the can of Coke and the can of Diet Coke are dropped into the fish tank full of water. Will they sink or will they float? Some students are surprised to find out that the Diet Coke will float while the regular Coke sinks. Be sure to check this out before you actually do it for your students because sometimes this demonstration does not work! It helps to use cold water and warm soda. The goal of this activity is to spark interest and provide a reference point for a discussion about density, which will follow as the lesson develops. At the end of the demonstration explain what the students will be doing in the lab.

2. **Group Roles**: Have the students work in groups of four. Discuss the group roles with the students. Each group should have a facilitator, a materials person, a time keeper, and a reporter. Each person will serve as grapher for one of the four sets of data. The materials person should gather everything that the group needs including the activity sheets.

3. **Data Collection**: Students work in groups and follow instructions for collecting the data. Remind students to read the graduated cylinder from the bottom of the meniscus and to place the graduated cylinder on a level surface when the reading is made. In each group, two students collect data using liquid A (water), and two students collect the data using liquid B (oil). They first weigh the graduated cylinder; then they add about two cubic centimeters of liquid and weigh the graduated cylinder and the liquid. After recording both the mass and the volume, the students repeat this process until they have completed six trials. When the data has been recorded, the students complete columns 4 and 5 on the data table. At this point each group of four has two data tables—one for liquid A and one for liquid B.

4. **Data Patterns**: Lead a discussion with the entire class to verify the data patterns and check the work of the groups. The students need to see that in column 4 the values decrease, and in column 5 the values are closer to a constant value. Students should also see that for liquid A, the values in column 5 are all close to 1, and for liquid B, the values in column 5 are all less than 1. Have each group determine the median value for column 5 for liquid A and for liquid B, and list these on the board.

5. **Scatterplots**: Students should follow the instructions for creating the scatterplots and determining the equations for the lines. The students working with liquid A should use a red pen, and the students working with liquid B should use a blue pen, as they create their graphs. Be sure to remind the students to plot all of the graphs on the same sheet of graph paper and to share the work, making sure that each student completes one scatterplot and line of best fit.

6. **Linear Equations:** After they have created the scatterplots and drawn the lines of best fit, the groups need to determine the equations for the lines. They need to remember that they can calculate the slope by determining the change in the y-values divided by the change in the x-values for any two points on the line. Using the slope and the y-intercept, the students should write the equation for the line using the form $y = mx + b$, where m is the slope of the line, and b is the y-intercept. By the time the groups finish this part of the lab, they should have the four scatterplots and lines drawn on the graph paper and the equations for these lines recorded next to each line on the graph.

7. **Scatterplots and Linear Equations Using the Calculator:** Each student should enter and graph all of the data. The data for liquid A from columns 1, 2, and 3 of the data table should be entered into L1, L2, and L3; and the data for liquid B from columns 1, 2, and 3 should be entered into L4, L5, and L6. Each student should graph the scatterplot and use the calculator to determine the linear equation for the data sets: (L1, L2), (L1, L3), (L4, L5), and (L4, L6). Students should compare their answers and record the linear regression equations determined by the calculator on the graph paper next to the corresponding lines. Remind the students to round off to two decimal places. Students should verify that the linear models they calculated by hand are close to the equations determined by the calculator.

8. **Data Interpretation**: Have each group display their graphs and linear equations. Students should discuss the answers to the interpretation questions within their groups. They should see from their graphs that the two red lines are parallel to each other, and that the two blue lines are parallel to each other, with the slopes of the red lines being greater than the slopes of the blue lines. Pull the groups together for a class discussion. It should be emphasized and brought out in the discussion that the slope of the line is the density of the liquid. Students should also realize that the y-intercept is the mass of the graduated cylinder. From the graphs, students can see that the density of liquid A is greater than the density of liquid B. Have the students pour the two liquids together and discuss what they see. Point out that mathematically they determined that liquid B was less dense than liquid A, thus verifying what was determined mathematically.

9. **Back to the Beginning**: Discuss what the results of the Mystery Liquids experiment have to do with the Coke/Diet Coke demonstration. Students should be able to conclude that the density of Diet Coke is less than one and therefore the slope of a Diet Coke line would be less steep than the line for liquid A. They should also be able to conclude that the slope for a Coke is greater than one and would be steeper than the line for liquid A.

## Assessment

Throughout this lesson there are opportunities to gain valuable insight into student understanding. What do the students know about density? What do they bring to the opening demonstration? Group work gives the teacher the opportunity to move from group to group to evaluate the general level of success that the students are having at a variety of different points in the lesson.

How well do the students do with data collection? Do they understand how to fill in the data chart and make the required calculations? When students report the ratio of mass to volume for liquid A and liquid B, not only is the teacher able to quickly determine where the class is at that point, but also the students are able to determine where they are compared to the rest of the class. In the video class, most groups are able to affirm their work by seeing that other groups had similar results. In one group where this is not the case, the students are able to evaluate their thinking and make some corrections.

Are the students able to make the scatterplots and determine the linear equations, both by hand and with the graphing calculator? Assess these skills by watching the students as they work in groups, and also ask questions of the students both to direct their thinking and to evaluate their understanding.

Group reports toward the end of the class give the students the opportunity to share their work both visually and orally with the entire class. Again, the teacher can evaluate students' understanding while the students have the opportunity for self-assessment. Pulling the lesson together by making sure that students discuss the key issues is a powerful means of assessing student understanding.

The homework assignment provides an opportunity to evaluate the written work of students to determine what they have learned from the lesson. Through written work, oral presentations, group work, and class discussions, student thinking can be assessed. Although most of these means of evaluation will be used for the purpose of guiding instruction, the written work and the oral presentations might also be evaluated for grading purposes.  If grades are to be assigned to either the homework or the oral presentation, having students know the criteria in advance is very important.

## Extensions & Adaptations

- This lesson does not depend on the graphing calculator, although use of the calculator does provide nice practice and a good check on the work the students do by hand.  Item 7 in the procedure could be skipped.

- While appropriate for any class studying linear models for the first time, this lesson would provide thoughtful review for a more advanced class.

- The concept of density is an important one for science students. Check with your science teachers for other sets of density data, or have students collect data and calculate the density of other materials.

- An extension for the students: Direct variation is described as a situation in which y varies directly as x or where y is directly proportional to x, and is expressed as $y = kx$ where k is any non zero number. Do any of the lines on the graphs illustrate this relationship? Defend your position.

- While you might be inclined to try calculating the density of Coke and of Diet Coke, attempts to do this have not been very successful. The problem is that the densities of Coke and of Diet Coke are both very close to 1 and precise measurements are difficult make.

## Mathematically Speaking

Linear functions are fundamental to the study of algebra. Students need practice in recognizing scatterplots that are linear; they need to be able to draw an eye-ball line of best-fit; and they need to be able to determine an equation for a line. The fundamental property of linear models—a constant rate of change—should be explored through tables, graphs, and symbolic representations.

Many students may have noticed that on the graphing calculator there are two forms of the linear regression model, LinReg(ax+b) and LinReg(a+bx). The first model is similar to $y = mx + b$, but the second might be a more helpful model for some students. If students graph $y = a + bx$, they start at the y-intercept a, and then move over 1 unit to the right and then b units in the vertical direction. These two forms use both the slope and the y-intercept, which are fundamental concepts dealt with in linear functions. This lesson helps students understand that there is a relationship between those characteristics of the algebraic model and the physical situation that the model represents.

Are all linear functions direct proportions? Some students are under the misconception that this is true. A direct proportion, for example when y varies directly as x, is represented by the equation $y = kx$, where k is the constant of proportionality. In the video lesson, the lines representing the ratio of mass to volume without the mass of the graduated cylinder are direct proportions, and they should pass through the origin. Direct proportion is an important concept in both mathematics and science classes.

A final point to highlight is the use of independent and dependent variables. The students need to understand that in this case the volume is the independent variable and mass is the dependent variable. The independent variable, or the volume, is plotted on the x-axis, and the dependent variable of the mass is plotted on the y-axis. Using these terms should also help students make connections between mathematics and science.

## Tips From Ellen

### Cooperative Learning

Group work is not necessarily cooperative learning. One difference is that cooperative learning avoids the chauffeur/hitchhiker phenomenon by establishing and balancing both a sense of positive interdependence (we sink or swim together) and individual accountability among team members. Here are some tips for promoting positive interdependence:

| | |
|---|---|
| ***Limit resources*** | Provide only one recording sheet, ruler, piece of graphing paper, marker, or measuring device per team or pair within a team. |
| ***Jigsaw information*** | Distribute the information so that each team member receives only a portion and the team must collaborate for all to understand the material. |
| ***Divide the task*** | This is similar to jigsawing information, except that a complex task is subdivided into several interrelated tasks. |
| ***Assign roles*** | Typical useful roles involving group investigations are materials manager, recorder, reporter, timekeeper, leader, and facilitator. Roles should be taught and assigned to suit the task. |
| ***Build a sense of team*** | Create team names and team handshakes, and do other team building activities that bond students. |
| ***Create one product*** | Either have group members collaborate on one product which will be shared and/or collected, or encourage each team member to create a quality product because only one will be collected randomly to represent the team. |
| ***Provide recognition or rewards*** | This positive reinforcement can be as simple as applause or can include free homework passes or bonus points. |

At the same time, all team members must know that they are individually accountable to contribute to the group and to learn. This can be accomplished using some of the same methods as above: jigsawed information, task division, and roles. It can also be accomplished by traditional methods such as informal and formal individual assessments.

Creating teams of optimal size and arranging the physical space to suit group work is also critical. Optimal size may mean anywhere from two to six persons, depending on the task and the size of the group. Groups of four tend to work well for most group investigations. This size is large enough to promote lively discussion of a complex problem, but not so large as to allow students to avoid involvement. Groups of four can also subgroup temporarily into pairs. Notice that given the desk arrangement for this lesson, students were turned sideways so that no student had his or her back to the front of the room.

### Cooperative Learning and Assessment

This lesson aligned the teacher's goals, instructional techniques, and assessment. The teacher was strongly interested in not just what students learned, but also how well they were able to communicate to others what they learned and what their thought process was. The instructional techniques of small group problem-solving and whole group discussion promoted oral communication. The assessment validated the importance of the group work by incorporating the group project as part of the grade, but also required a journal to assess individual learning and written communication skill.

Group grades should be used with caution. To paraphrase Spencer Kagan, an international consultant in cooperative learning, any grading method must be questioned when two students with the same ability, the same motivation, the same instruction, and the same effort, are likely to receive different scores; but different scores are likely to occur if these students are in different groups. If group projects are assessed, the results might be used for team recognition rather than grades, or if part of a grade, should not be high stakes in nature.

## Resources

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 9 - 12 : Algebra in a Technological World*. Reston, Virginia: National Council of Teachers of Mathematics, 1991.

Internet location: **http://www.mtse.uiuc.edu/regression/titlepage.html**
This is an excellent site that gives step-by-step instructions for finding a linear regression equation. It also has two data files that can be downloaded.

Internet location: **http://www.glenbrook.k12.il.us/gbsmat/travel/bonvoyage.html**
Currency exchange rates, mathematics, and the internet are woven together to allow students to investigate currency exchange rate of different countries and create a report. Student handouts, grading rubric, and internet links are included.

Internet location: **http://www.cs.rice.edu/~sboone/Lessons/Titles/lphouse.html**
This site has students collect housing prices over the internet and then develop a linear model for the relation between the cost of the house and the number of square feet. From there, students will interpolate and extrapolate the cost or size of any house.

Internet location: **http://www.cs.rice.edu/~louviere/Lessons/les6.html**
"It's a *Matter* of Density" is a lesson that will allow students to further explore the concept of density. The lesson is complete with data tables, questions, links, and a scoring rubric.

Internet location: **http://www.glenbrook.k12.il.us/gbsmat/regress/regress.html**
This internet project leads students through determining a linear regression model on the TI-82 calculator.

Internet location: **http://www.math.psu.edu/OtherMath.html**
This site offers a collection of resources from Penn State.

## Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

**Standard 1: Worthwhile Mathematical Tasks**

1. Discuss the importance of the content of this video lesson. How does it help students with their algebraic concepts, as well as with the science topics that were covered? Students sometimes do not relate what they learn in the mathematics classroom to what they learn in the science classroom, so integrated lessons such as this one are important because they help students make those connections. What other connections between mathematics and other curriculum areas do you use for integrated lessons?

**Standard 2: The Teacher's Role in Discourse**

2. Identify ways that the video teacher had students clarify ideas and concepts. What techniques do you use in your classroom to draw input and knowledge from students?

**Standard 3: Students' Role in Discourse**

3. Discuss the role of student discourse in this lesson. Was the discourse critical to the lesson? Were all students involved? Was there gender equity? How do you handle these issues in your classes?

4. Did you feel the students in this lesson were comfortable disagreeing with each other? How did this affect discourse? What can you do to promote this type of openness in your classroom?

**Standard 4: Tools for Enhancing Discourse**

5. What tools and concrete materials were used in the Mystery Liquids lesson to enhance discourse, and how effective were they? What are advantages and disadvantages of group reports such as those at the end of the video lesson where the students shared their graphs?

**Standard 5: Learning Environment**

6. At the beginning of the lesson, the video teacher used the demonstration with the fish tank and Coke cans to capture students' attention and spark their interest. How effective was this? What are some of your favorite ways to launch a lesson?

**Standard 6: Analysis of Teaching and Learning**

7. How did this video teacher analyze teaching and learning? What do you do to efficiently monitor student progress?

8. As teachers assess student learning in a variety of ways, especially ones that involve evaluating written responses, teachers are confronted with the challenge of efficiently handling paperwork. What do you do to manage this concern?

9. How did the teacher adapt instruction based on student responses? How do you balance changing a lesson to insure student understanding with the need to complete the lesson in the given timeframe?

# MYSTERY LIQUIDS

In your group of four, two students will collect data on liquid A, and two will collect data on liquid B. Share your data within the group to complete both data tables. When measuring, have your partner check your measurements.

## Part 1: Collecting Data

1. Find the mass of an empty graduated cylinder.
   Record the mass to the nearest 0.01 gm at the top of the data table.

2. Half of your group should collect and record data in a data table for liquid A, and the other half should collect and record data in a separate data table for liquid B. Record which liquid you are analyzing at the top of the data table. Pour about 2 $cm^3$ (ml) of your liquid into the graduated cylinder. Record the exact volume of the liquid in column 1 (trial 1) in the data table (to the nearest 0.1).

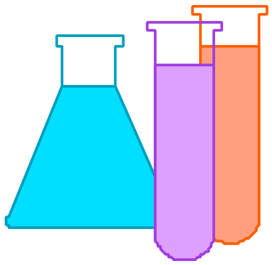
3. Find the mass of the system (graduated cylinder containing the liquid). Record this mass in column 2 of your data table. Find the mass of the liquid alone and record this value in column 3 of your data table.

4. Add about 2 additional $cm^3$ of the same liquid to the graduated cylinder. Again record the exact volume of the liquid now in the graduated cylinder in column 1 (trial 2). Repeat step 3 and again record both masses in columns 2 and 3.

5. Repeat step 4 until you have completed six trials.
   Record the data after each trial.

6. Complete columns 4 and 5 of the data table.

When you have completed collecting data, you should have two data tables completed in your group—one for liquid A and one for liquid B.



# Mystery Liquids
## Data Table
## for Liquid _____
(specify liquid A or B)

**Mass of empty graduated cylinder __________**

| Trial | Column 1<br>Volume of Liquid | Column 2<br>Mass of Cylinder plus Liquid | Column 3<br>Mass of Liquid | Column 4<br>Ratio of Column 2 to Column 1 | Column 5<br>Ratio of Column 3 to Column 1 |
|---|---|---|---|---|---|
| 1 | | | | | |
| 2 | | | | | |
| 3 | | | | | |
| 4 | | | | | |
| 5 | | | | | |
| 6 | | | | | |

## Part 2: Graphing Data Manually and Finding Equation

On one large sheet of graph paper, graph the following sets of data:

**From Data Table for Liquid A:**

1. Using a red pen, graph data from column 1 and column 2 for liquid A. Which column should represent the independent axis? Which one are you controlling? Draw an approximate best-fit line. Find the equation of this line algebraically. Show your work below.

2. Using a red pen, graph data from column 1 and column 3 for liquid A. Draw an approximate best-fit line. Find the equation of the line algebraically. Show your work below.

**From Data Table for Liquid B**:

3. Using the same sheet of graph paper that you did for Liquid A and a blue pen, graph data from column 1 and column 2 for liquid B. Which column should represent the independent axis? Which one are you controlling? Draw an approximate best-fit line. Find the equation of the line algebraically. Show your work below.

4. Using a blue pen, graph data from column 1 and column 3 for liquid B. Draw an approximate best-fit line. Find the equation of the line algebraically. Show your work below.

## Part 3: Using a Calculator to Graph and Determine the Linear Model

Each person should enter and graph all data.

### *Liquid A*

1. Enter the data from the data table for liquid A in L1, L2, L3.

2. Graph a scatterplot of data from columns 1 and 2. (L1, L2).

3. Determine the best-fit line by calculating the linear regression model for the (L1, L2) data. Graph the line on the calculator. How good a fit is the line? Record the linear equation on your graph paper, rounding to two decimal places.

4. Turn off all scatterplots and equations. Graph a scatterplot of the data from columns 1 and 3. (L1, L3).

5. Determine the best-fit line by calculating the linear regression model for the (L1, L3) data. Graph the line on the calculator. How good a fit is the line? Record the linear equation on your graph paper rounding to two decimal places.

### *Liquid B*

1. Enter the data from the data table for liquid B in L4, L5, L6.

2. Turn off all scatterplots and equations. Graph a scatterplot of the data from columns 1 and 2. (L4, L5).

3. Determine the best-fit line by calculating the linear regression model for (L4, L5) data. Graph the line on the calculator. How good a fit is the line? Record the linear equation on your graph paper rounding to two decimal places.

4. Turn off all scatterplots and equations. Graph a scatterplot of the data from columns 1 and 3. (L4, L6).

5. Determine the best-fit line by calculating the linear regression model for the (L4, L6) data. Graph the line on the calculator. How good a fit does it seem to be? Record the linear equation on your graph paper rounding to two decimal places.

## Part 4: Interpreting the Data

1. Compare the graphs for liquid A with the graphs for liquid B. How are they similar? How are they different?

2. Compare the slopes of the two lines for liquid A. Do the same for liquid B. What could be the reason for your observation?

3. Compare the slopes representing liquid A with the slopes representing liquid B. What could be the reason for your observation?

4. Explain the physical meaning of the y-intercepts.

5. What does the slope represent?

# Directions for Using the TI-82 to...

## Graph a Scatterplot; Determine a Linear Model; and Graph the Model

**To enter data:**

**STAT** **1:EDIT** **ENTER**
(Clear all lists by using the up arrow button to go to the top and pushing clear.)
Enter data with L1 the independent variable and L2 the dependent variable.

**To graph a scatterplot:**

**2nd** **Y (STAT PLOT)** **1:PLOT1** **ENTER**
Plot on; type: scatter
X list: L1; Y list: L2; Mark: 1st symbol.

**To choose window and graph**:

**ZOOM** **9:ZoomStat** **ENTER**

| YOU SHOULD SEE A SCATTERPLOT. |
|---|

**To find linear regression:**

**STAT** **CALC** **5:LinReg(ax+b)** **ENTER**

**To graph the line**:

**Y=** **VARS** **5:Statistics** **EQ** **7:RegEQ** **GRAPH**

| YOU SHOULD GET A LINE THOUGH THE SCATTERPLOT. |
|---|

# *Selected Answers*

## Part 4: Interpreting the Data

1. Compare the graphs for liquid A with the graphs for liquid B. How are they similar? How are they different?

   *The graphs for liquids A and B are all straight lines. The pairs of lines have different slopes, but the same y-intercept.*

2. Compare the slopes of the two lines for liquid A. Do the same for liquid B. What could be the reason for your observation?

   *Both A lines have the same slope and both B lines have the same slope. This makes sense because the slope of the line represents the change in mass divided by the change in volume. In one case the mass of the graduated cylinder is included, and in the other case this mass has been subtracted.*

3. Compare the slopes representing liquid A with the slopes representing liquid B. What could be the reason for your observation?

   *The slope of the A lines is about 1, and the slope of the B lines is less than 1. This is because liquid A (water) has a density of 1, and liquid B (cooking oil) has a density that is less than 1.*

4. Explain the physical meaning of the y-intercepts.

   *The y-intercept is the mass of the graduated cylinder. If there is no liquid in the graduated cylinder, then the y-intercept gives the mass.*

5. What, if anything, does this lab have to do with density?

   *The slope of the lines represents the densities of the liquids. Density is mass divided by volume which equals slope.*