PBS Mathline®

Pennies, Pressure, Temperature, and Light

(Inverse Functions)

Objective

The major goal of this lesson is to collect data from a variety of experiments, and then determine what type of model best fits the data, and explain why. Students will explore a variety of relationships using pennies, pressure, temperature, light, and pendulums to determine the algebraic equation that best represents the pattern modeled by the variables involved in each situation.

Overview of the Lesson

The lesson begins with a review of the cooling curve and a discussion of the algebraic model that best represents that data. The students are then assigned five experiments involving distance and number of pennies that can be balanced, pressure and volume, distance and light intensity, mass and distance from the center in a see saw situation, and what determines the period of a pendulum. The discussions generated by these experiments demonstrate the students' strong background in functions and are a very important part of this video. Two of the five experiments are discussed in the video, but all five are included in the written lesson guide.

Materials

- graphing calculator overhead unit
- overhead projector

For *Newton's Law of Cooling*
- a cup of very hot water
- CBL with temperature probe
- graphing calculator with link
- *Newton's Law of Cooling* activity sheet

For *Light Intensity*
- CBL with light probe
- graphing calculator with link
- simple car made with a piece of 2x4 with wheels
- light source
- *Light Intensity* activity sheet

For *The See Saw Experiment*
- a fulcrum (could be the back of a chair)
- meter stick
- string
- masses
- *The See Saw Experiment* activity sheet

For *Penny Functions*
- a balance scale (optional)
- centimeter ruler
- 10 pennies for each group
- *Penny Functions* activity sheet

For *Boyle's Law*
- CBL
- graphing calculator with link
- Vernier pressure sensor
- *Boyle's Law* activity sheets

For *The Pendulum Problem*
- a long cord tied around a heavy ball
- masses
- stop watch
- *The Pendulum Problem* activity sheet

Procedure

1. **Introduction**: In the video lesson, the teacher reviews exponential models with the students by doing the *Newton's Law of Cooling* experiment and discussing the results. This is followed by groups working on five different experiments. In this lesson guide, the *Newton's Law of Cooling* experiment is listed as one of the activities. Each of the six activities is listed and discussed

separately so that you can modify the procedure to meet the needs of your students. For example, if your students have done extensive data analysis work, you may wish to assign a different experiment to each group and have the groups report to the class on their findings emphasizing various function models. For other classes, you may wish to have the entire class do just one experiment, or several groups within the class all do the same experiment, focusing on one function model.

2. **Newton's Law of Cooling**: Assuming this is a review, begin by asking the students whether they should expect to get the same cooling curve using water as they got when they used aluminum foil. Have students examine the exponential model $y = a(r)^x + b$, where y represents temperature and x represents time. Be sure that your discussion brings out the point that a represents the starting temperature, and b represents the room temperature. Also discuss the fact that the value for r is related to the chemical composition of the foil or the water.

   Next, collect cooling curve data using the CBL with the temperature probe inserted in a cup of hot water. The time and temperature data are loaded into the graphing calculator by the CBL, and a scatterplot is displayed for the class. Have the students subtract the room temperature from the function values so that the curve will be asymptotic to the x-axis. Once this translation is done, the data is in a form that can be used to do a fit with the graphing calculator. Students use the calculator to derive and test several models, and they use residuals to determine whether the model is the correct one for the data set. A residual is the difference between the actual value of the data point and the value for the function. If the residuals are good, they will have no pattern and a small window. A pattern in the residuals indicates that students are using the wrong family of functions. After the calculator gives an exponential model, modify that model by adding the constant value of the room temperature. After completing this review, students are ready for the new activities.

3. **Light Intensity**: After setting up the experiment and making sure that the program functions properly, instruct the group to move the car so that the scatterplot of *Separation Distance* vs. *Time* looks like a cubic model. Student groups must determine that they have to move the car away from the light, back toward the light, and then away again. Once the groups use the CBL connected to the light probe to collect the data, students should display the data using the graphing calculator. Students look at two plots: *Light Intensity* vs. *Time*, *and Separation Distance* vs. *Time*. In the discussion of this experiment, help students understand that the two plots seem to be reflections of each other and that there is an inverse relationship between light intensity and separation distance.

Next students should try to determine the type of inverse relationship represented. Does the intensity vary inversely as the distance from the light source, or does the intensity vary inversely as the square of the distance from the light source? They set up a table to determine if $(x)(y)$ is a constant, which would mean the intensity varies inversely as the distance, or if $(x^2)(y)$ is a constant, which would mean the intensity varies inversely as the square of the distance.

When neither of these models seems to be quite right for the data set, suggest using the calculator to determine a power regression model. Have students determine the model; they will find that the intensity varies inversely as something that is actually between x and x^2.

4. **The See Saw Experiment:** The purpose of this experiment is to determine the relationship between the masses suspended from a meter stick and their distances from the center of the meter stick when the system is in balance. First, have students balance a meter stick on a fulcrum. Then have them use string to suspend a 200 gram mass at one end of the meter stick. Next, students should suspend a 500 gram mass on the other side of the fulcrum, moving it until they reach the point at which the system is in balance. Students then repeat this using 1000 and 300 grams. Students then plot the data and determine the equation that best represents the relationship. They find that an inverse relationship exists.

   Students then consider a second problem. If they place a 100 gram mass at the opposite end of the meter stick from the 200 gram mass that stays motionless, where on the 100 gram-side would they have to place a 500 gram mass to have the system in balance? Do not perform this experiment in class; rather, assign it as part of the students' homework. If students need help, hint that they may determine the balance point for the side with the 100 and 500 gram masses, and then use that as the position for the 600 gram mass on that side of the fulcrum.

5. **Penny Functions**: In this experiment, students explore the relationship between the number of pennies on the end of a ruler and the distance the ruler can hang over the edge of a table. They place a ruler on the edge of a level table with the end marked zero extending over the edge. Then they place one penny at the last mark on the other end of the ruler. Using a pencil, they push the ruler toward the edge until it starts to tip over noting how far the ruler extended over the edge. This process is then repeated using from 2 through 10 pennies. Students record the data in a table as they complete the experiment. After the data has been collected, the students plot the data and answer various questions concerning the relationship.

6. **Boyle's Law**: Before the students actually conduct this experiment, they are asked to make a conjecture about the relationship between the volume of gas in a container and the pressure it exerts on the container. They then set up the equipment, attaching a short piece of tubing at the end of the hypodermic to the pressure sensor and attaching the CBL to the graphing calculator and the pressure sensor. After opening the release valve, students execute the program *Pressure*, hitting the "Enter" key to zero the pressure. The students continue to follow the written instructions to collect the data. They make a scatterplot and study it to determine what type of function this data represents. Students must recognize the pattern of the inverse function and then use the power model regression on the graphing calculator to determine the equation.

7. **The Pendulum Problem**: In this investigation, students determine what controls the period of the pendulum, and they also find the algebraic equation that represents that relationship. Three factors are explored as the students seek to understand the relationships among the variables:

   - first, by holding the length of the string and the arc constant, they explore the relationship between the period of the pendulum and the mass suspended;
   - second, by keeping the mass and the arc constant and varying the length of the string, they investigate the relationship between the period and the length of the pendulum;
   - third, by holding the mass and the string length constant, they investigate the relationship between the period and the size of the arc.

   After collecting the data for each of these relationships, the students examine their findings and determine that the mass and the arc do not change the period, but the length of the string does. The period is equal to some constant times the square root of the length of the string.

8. **Student Presentations and Class Discussion**: After the students complete their experiments, have two groups make presentations. This provides an opportunity to clarify the experiments and the results of the experiments, as well as to link their current work to work that was completed earlier in the year. Encourage the students to see connections in a variety of different ways and to use data analysis to explore families of functions.

Assessment

Throughout these experiments, student discussions of families of functions are a very important part of assessing student understanding. Ask students many questions about the observations and calculations that they are making throughout the lesson. Such questions not only help to lead the students, but they also help the teacher evaluate student understanding.

Assessment based solely on observing students determining a correct answer might differ significantly from assessment based on students demonstrating a deeper level of understanding by explaining orally or in writing how they got the answer and how they know that it is correct. This is highlighted in the video, and it brings out the importance of spiraling important concepts throughout the year. The development of the concept of function families for this video teacher begins in the fall and continues throughout the entire year. Through assessment at a variety of levels, the teacher is able to determine when mastery has been reached and tailor instruction based on those observations.

Extensions & Adaptations

- Students could do some of these experiments, or explore some of the patterns using data supplied in the lesson plan, in connection with the lesson *Getting Out of Line* (HSMP). Students could match the data patterns they find from these experiments to the patterns they noticed in *Getting Out of Line*. Emphasize creating scatterplots, looking for patterns, and trying to categorize the patterns.

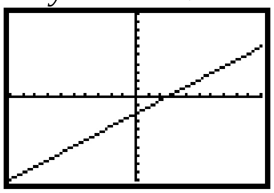
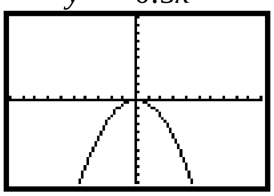
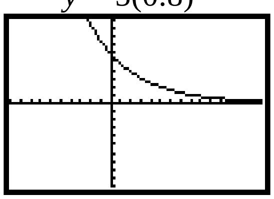
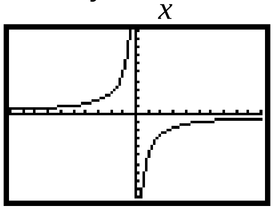
- Use each experiment as a separate lesson. Use small groups whenever possible, but also have the entire class discuss the experiment completely.

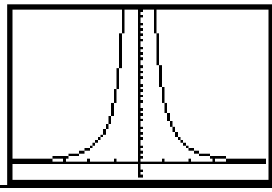
- *Penny Functions* gives data that cannot be modeled well by any of the calculator models. The activity sheet included in this lesson guide does not ask students to determine a mathematical model. However, this could be an extensionof the activity. In order to determine the model, students have to think about the physics of the lever and derive the model algebraically. A discussion is included in the answer section for *Penny Functions* activity.

Mathematically Speaking

As students grow in their understanding of algebra, they develop their knowledge of function families (the characteristics of the tables, graphs, and symbolic form of those functions) and their ability to describe the characteristics of functions, both orally and in writing.

The following table contains examples of some of the basic functions and words or phrases that describe those functions. As students continue their study of high school mathematics, they will add to this list trigonometric, logarithmic, greatest integer, rational, and polynomial functions. Throughout their study of mathematics, as students revisit previously studied functions and explore new function models, they need to focus on variables, functions, and families of functions.

Graph	Words or Phrases that Describe the Pattern
Linear Function $y = -2 + 0.8x$ 	line linear model constant rate of change x-intercept of 2.5 y-intercept of –2 as x becomes very large, y is very large as x becomes very small, y is very small defined for all values of x
Quadratic Function $y = -0.5x^2$ 	passes through the origin maximum value of 0 for $x > 0$, as x increases, y decreases for $x < 0$, as x decreases, y decreases $y \leq 0$ symmetrical about the y-axis rate of change is not constant concave down defined for all values of x
Exponential Function $y = 5(0.8)^x$ 	$y > 0$ y-intercept of 5 as x becomes very large, y gets closer to 0 the x-axis is a horizontal asymptote as x becomes very small, y becomes very large rate of change is not constant exponential defined for all values of x concave up
Power Function (Inverse Model) $y = \frac{-2}{x}$ 	as x becomes very large or very small, y approaches 0 as x approaches 0 from the positive side, y becomes very, very small as x approaches 0 from the negative side, y becomes very, very large y-axis is a vertical asymptote x-axis is a horizontal asymptote curve has two branches symmetrical to the lines $y = \pm x$ symmetrical about the origin rate of change is not constant concave up for $x < 0$; concave down for $x > 0$; not defined for $x = 0$

Power Function (Inverse Square Model) $y = \frac{10}{x^2}$ 	as *x* becomes very large, or very small, *y* approaches 0 the *x*-axis is a horizontal asymptote as *x* approaches 0 from the positive or negative side, *y* becomes very large the *y*-axis is a vertical asymptote curve has two branches $y > 0$ symmetrical to the *y*-axis rate of change is not constant concave up and defined for $x \neq 0$

As students explore and describe these functions, the following questions might help them think about characteristics that should be considered.

Are the values of *x* (or *y*) increasing or decreasing?
Is there a maximum (or minimum) value?
Is the graph defined for all values of *x*?
What is the shape of the graph?
What is the rate of change? Is it constant? Increasing? Decreasing?
Are there *x*-intercepts? If so, what are they?
Is there a *y*-intercept? If so, what is it?
As *x* becomes very, very large (or small), what happens to *y*?
Does the graph pass through the origin?
Is the graph symmetrical? To a line? To the *y*-axis? About a point?
Is the graph curved up (concave up) or curved down (concave down)?
Are there asymptotes?

Tips From Ellen

Addressing Change

Classrooms that address reform mathematics look very different from classrooms of not too long ago. As teachers address the changes in how they teach, they do not have the same models on which to rely. Nothing can be taken for granted. Problem sets no longer provide the type of practice that is needed. Old tests and exams become outmoded. Tutors and parents may not be equipped to serve as resources. Instruction has to be resequenced, with some old favorite topics and activities giving way to new. The things that we were good at, that were comfortable, may no longer have a place in the reform classroom.

Change becomes more difficult when there is little to model after. When you go on a journey, even if you are in unfamiliar territory, you can make progress if you have a strong detailed map with well defined landmarks. When you go into uncharted territory, you need to take with you a strong sense of direction or you may lose your way, and a strong sense of resolve or you will become discouraged and give up.

The Pinellas County School System in Florida has a metaphor for change that is useful. They contend that their education system was like a good train system. They became very good at moving lots of people on tracks to predetermined destinations. They had comfortable and efficient cars for that purpose. However, they found that they had many more people to move than could fit on their trains, and that they needed to take them to places for which there were no tracks and to go faster than trains could go. What they needed was a plane system, one that they couldn't even quite envision. The problem was that they couldn't shut down the whole system while they fully designed and changed over from a train to a plane system. Some people said that they shouldn't even try—they were good at trains and should stick to what they knew. But others became convinced that they would be out of business if they ignored the need for planes. So they have agreed to acknowledge the difficulty of changing systems, but that they will no longer lay train tracks and will buy only plane parts. They have direction and resolve.

The teacher portrayed in this video has a strong sense of direction and resolve. He has internalized for himself a map of what it means to understand the families of functions that are the heart of his subject. His spiral approach to instruction always gives students familiar ground on which to rest while tantalizing them with uncharted territory yet to be mapped, and instills in them the confidence that he knows where they are going and that they will not become lost.

Resources

CBL™ System Experiment Workbook. Dallas, TX: Texas Instruments, 1994.

CBL™ Explorations in Algebra for the TI-82 and TI-83. Erie, PA: Meridian Creative Group, A Division of Larson Texts, Inc., 1996.

Brueningsen, Chris; Bill Bower, Linda Antinone, and Elisa Brueningsen. *Real-World Math with the CBL™ System*. Dallas, TX: Texas Instruments, 1994.

Internet location: **http://www.ti.com** This site offers various calculator programs, CBL programs for the calculator, lesson activities, and resources for teachers that can be downloaded from the web site. There is a great deal of material available. There are also links to other mathematics sites.

Smith, Stanley A., and Randall I. Charles. *Addison-Wesley Algebra Enrichment*. Reading, MA: Addison-Wesley Publishing Company, 1988.

Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

Standard 1: Worthwhile Mathematical Tasks

1. The video teacher in this lesson uses a strong functions approach in teaching algebra. What are the advantages of this approach over a traditional algebra program? What do you do in your classroom?

2. How does the CBL enhance this algebra lesson? Do you use CBL's in your classroom? If not, which lab would be a good lab to try first?

3. *Penny Functions* has multiple levels, and ties in to *The See Saw Experiment* activity. How would you use this activity with your students?

Standard 2: The Teacher's Role in Discourse

4. The video teacher states that he wants a relaxed atmosphere, one where the students feel free to call out answers. This seems to work very well for him. What kind of atmosphere do you like to have in your classroom? What do you do promote that atmosphere?

Standard 3: Students' Role in Discourse

5. In this lesson, students were involved in whole-class discussions led by the teacher, and in small group discussions sometimes with the teacher and sometimes with just other students. How did the students function in these different situations? What types of discussion forums do you have in your class, and what do you expect from your students? How do you convey these expectations?

Standard 4: Tools for Enhancing Discourse

6. CBL's offer rich opportunities for doing data analysis labs that were not possible until now. How should these labs be organized? Is it enough to do a demo like the video teacher did in this lesson when collecting the cooling curve data, or is the actual use of the CBL important enough to have small groups of students using one? How do you handle this in your classroom?

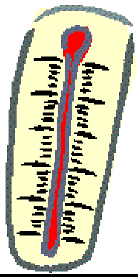
7. With mathematics teachers using more and more labs in math classes, there is a need to use more lab equipment. This offers opportunities for math teachers who need to borrow equipment to talk to science teachers. What is happening in your school? Do you share equipment? Are you talking more because of this? If you are currently not communicating with your science department, what are some ways to begin a dialogue?

8. Teachers all hope that students are not using the calculator for simple calculations that they should be able to do in their heads. What observations have you made concerning calculator use in your classroom? Are students using the calculator for simple operations? If so, how can you help students recognize the importance of estimation and number sense so that they will use the calculator most effectively?

9. In the video lesson, the teacher has difficulty with the calculator program while collecting data for the light intensity experiment. He is able to fix the program and continue the lab. However, he is not able to fix the syringe for the pressure experiment. When using technology and equipment that is sensitive, there is always the possibility of technical difficulty. What are some alternatives plans that can enable a lesson to be effective even when technical difficulties occur? For some teachers, these sorts of incidents are enough to make them not want to use these labs. Have you ever had a lab fail because of technical difficulties? What did you do?

Standard 6: Analysis of Teaching and Learning

10. The video teacher mentions writing up the lab in a lab book, and from time to time in the lesson tells students to be sure to include certain points in their lab report. Do you have your students write up lab reports for every data analysis lab that they do? It is important to establish grading criteria when students write lab reports. What type of rubric do you use?



Newton's Law of Cooling

This experiment allows students to investigate the temperature variations of a cooling object.

Introduction

As soon as a hot cup of coffee is poured, it begins to cool. The cooling process is rapid at first, then levels off. After a long period of time, the temperature of the coffee eventually reaches room temperature. Temperature variations for such cooling objects were summarized by Newton. He stated that the rate at which a warm body cools is approximately proportional to the temperature difference between the temperature of the warm object and the temperature of its surroundings. Stated mathematically:

$$\frac{\Delta T}{\Delta t} = -k(T - C)$$

where ΔT represents the object's temperature change during a very small time interval Δt, T is the body's temperature at some instant, C is the surrounding temperature, and k is a proportionality constant. This equation can be solved using advanced techniques:

$$T - C = (T - T_0)e^{-kt}$$

where T_0 is the body's temperature when $t = 0$.

In this experiment, you will investigate temperature variations for a cooling object and attempt to verify the mathematical model developed by Newton.

Equipment Required

- CBL unit
- TI-82 graphics calculator with a unit-to-unit cable
- TI temperature probe
- Hot plate
- Medium beaker
- Laboratory thermometer
- Water
- Ice
- TI-Graph Link™ (optional)

Program Listing

This experiment requires that you download or enter the COOLTEMP.82P program listed.

```
PROGRAM:COOLTEMP.82P
:PlotsOff
:Func
:FnOff
:AxesOn
:0ÍXmin
:99ÍXmax
:10ÍXscl
:-20ÍYmin
:100ÍYmax
:10ÍYscl
:ClrList L2, L4
:ClrHome
:{1, 0}ÍL1
:Send(L1)
:{1, 2, 1}ÍL1
:Send(L1)
:99Ídim L4
:ClrHome
:Disp "PRESS ENTER TO"
:Disp "START GRAPHING"
:Disp "TEMPERATURE"
:Pause
:ClrDraw
:Text(4, 1, "TEMP(C)"):Text(54, 81, "T(S)")
:{3, 1, -1, 0}ÍL1
:Send(L1)
:For(I, 1, 99, 1)
:Get(L4(I))
:Pt-On(I, L4(I))
:End
:seq(N, N, 0, 98, 1)ÍL2
:0ÍXmin
:max(L2)ÍXmax
:10ÍXscl
```

```
:Plot1(Scatter, L2, L4,–)
:DispGraph
:Text(4, 1 "TEMP(C)"):Text(54, 81, "T(S)"
:Stop
```

Equipment Setup Procedure

To connect the equipment as shown by Figure 1:

1. Connect the CBL unit to the TI-82 calculator with the unit-to-unit link cable using the I/O ports located on the bottom edge of each unit. Press the cable ends in firmly.
2. Connect the temperature probe to Channel 2 (CH2) on the top edge of the CBL unit.
3. Turn on the CBL unit and the TI-82 calculator.

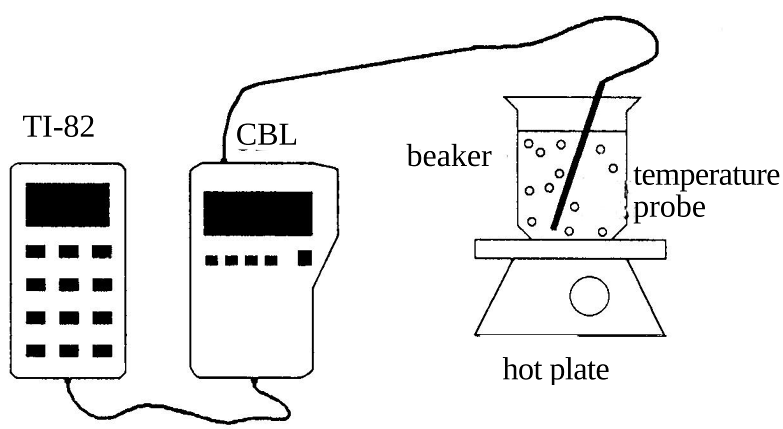


Figure 1: Equipment Setup

The CBL system is now ready to receive commands from the calculator.

Experiment Procedure

1. Read the laboratory thermometer to determine the room temperature in degrees Celsius and record this value as *C* in your lab notebook. Store this value in variable *C* on your TI-82.
2. Fill a medium beaker with water and place the beaker on a hot plate. When the water begins to boil, place the temperature probe in the beaker for several seconds.
3. Make sure the CBL is turned on. Start the program COOLTEMP on the TI-82. Remove the temperature probe from the boiling water, and press [ENTER] on the TI-82 to start collecting data.

The probe should remain exposed to the air while the CBL and TI-82 collect the temperature data. To avoid conduction and evaporation effects on the temperature probe, do not place the probe directly on the table top or expose it to any drafts.

4. Observe the resulting variations in temperature on the TI-82 display as the data is collected. Data is collected at a rate of 1 point per second for approx–imately 1.5 minutes. Temperature (in ˚C) is stored in L4 and time (in seconds) is stored in L2.

   Your graph should look similar to the one shown in Figure 2. After the CBL has finished collecting data, you may want to adjust Ymax and Ymin on the TI-82 to create an appropriate viewing window.

   (Press WINDOW on the TI-82 to access Ymax and Ymin.) When you are satisfied with the graph, save it to a PIC variable to be printed later using TI-Graph Link.

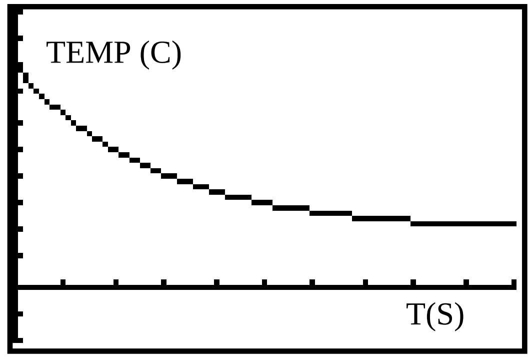


Figure 2: Temperature vs. Time

Analysis and Conclusion

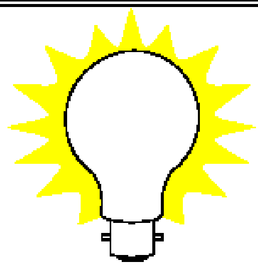
1. Print the PIC variable for this experiment using the TI-Graph Link (or make a sketch of your graph by hand), and attach it to your lab notebook. Be sure to include appropriate scales and axis labels on the print-out.

2. According to Newton's law of cooling, the quantity $y = T - C$ varies exponentially with time. To create a model of this relationship, you must first subtract room temperature from the collected temperature values. To do this on the TI-82, press **2nd** **[L4]** **-** **ALPHA** **C** **STO→** **2nd** **[L4]** **ENTER** on the home screen, where C is the room temperature value that you previously stored in your calculator.

3. Use your TI-82 to perform an exponential regression (ExpReg) on the collected data. Remember that the time data is stored in list L2 and the temperature data is stored in list L4. Record the regression equation and correlation coefficient in your lab notebook.

   Does the equation obtained using the TI-82 match the mathematical model relating temperature and time described in the introduction; i.e. , do temperature and time appear to vary exponentially?

Repeat this experiment at least two more times, and record all relevant data in your lab notebook. For one of the trials, start with the temperature probe in an ice bath, and allow it to warm up when removed from the bath. Is this relationship exponential? If it is, use the TI-82 calculator to find an appropriate equation for this data.



Light Intensity

This experiment allows students to determine a mathematical model for relationship between light intensity and the distance from the light source.

Introduction

What is the relationship between light intensity and the distance from the light source? In this experiment, you will derive a mathematical model from data collected while moving both a light probe and a motion sensor back and forth in front of a light source.

The sensors will collect data with respect to time. You will display the data (light intensity and distance) with respect to time, and then study the relationship that exists between light intensity and distance.

Equipment Required

- CBL unit (optional power adapter AC-9201 recommended)
- TI-82 calculator with a unit-to-unit cable
- TI light probe
- Vernier CBL motion detector (MD-CBL)
- Standard light bulb (25W)
- Large cardboard box (all inside surfaces painted with flat black paint)
- Wooden block (a 2 x 4 approximately 12 inches long)
- Tape
- Two rubber bands
- Small cart
- TI-Graph Link (optional)

Program Listing

This experiment requires that you download or enter the LIGHTDIS.82P and GETLIGHT.82P programs.

```
PROGRAM:LIGHTDIS.82P
:PlotsOff
:AxesOn
:Func
:FnOff
:{1, 0}Í L1
```

```
PROGRAM:GETLIGHT.82P
:ClrList L2, L3, L4
:Get(L3)
:Get(L4)
:Get(L2)
```

```
:Send(L1)
:{1, 11, 3} ÍL1
:Send(L1)
:{1, 1, 1} ÍL1
:Send(L1)
:{3, .1, 40,1, 0, 0, 0, 0,1}Í L1
:Send(L1)
:Stop
```

Equipment Setup Procedure

To connect the equipment as shown in Figure 1:

1. Connect the CBL unit to the TI-82 calculator with the unit-to-unit link cable using the I/O ports located on the bottom edge of each unit. Press the cable ends in firmly.

2. Tape the motion detector to the front of the wooden block. Be careful the tape does not cover any working part of the motion detector. Secure the TI light probe to the side of the wooden block with the two rubber bands. Place the CBL unit on top of the wooden block.

3. Connect the TI light probe to Channel 1 (CH1) on the top edge of the CBL unit. Connect the motion detector to the SONIC port on the left side of the CBL unit. Be sure all cables are firmly inserted and out of the way of the motion detector and the light probe.

4. Place the wooden block on top of the cart.

5. Remove the top of the box and place the box on its side, with the open side facing the cart. Place the light bulb inside the box. Make sure the box is large enough so that the motion detector's beam does not detect the sides of the box.

6. Tape a line (about three feet long) that is perpendicular to the back of the box from the light source along the top of a table. This tape line serves as a guide for the person performing the experiment to keep the light probe and motion detector constantly perpendicular to the box when the wooden block is moved.

7. Place the wooden block apparatus in front of the box, along the tape, about two feet away from the bulb. Align the middle of the bulb and the light probe so they are on the same level.

8. Turn on the CBL unit and the calculator. The CBL system is now ready to receive commands from the calculator.

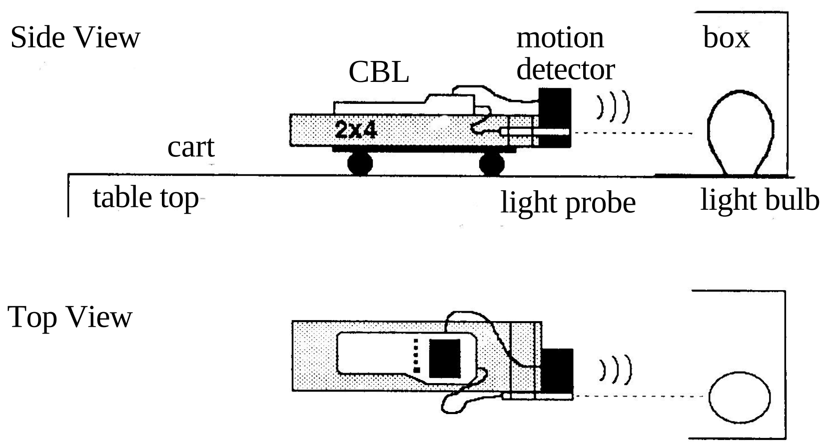


Figure 1: Equipment Setup

Experiment Procedure

1. Make sure the CBL is turned on. Start the program LIGHTDIS on the TI-82. The motion detector starts clicking. When the READY indicator is lit on the CBL display, disconnect the CBL unit from the calculator to allow the wooden block apparatus to roll more freely.

2. Darken the room, and turn on the light bulb.

3. Check the alignment of the wooden block. When the light probe is directly in front of the light source and the motion detector is aimed so that its beam is detected by the back of the box, press TRIGGER and start rolling the wooden block (on its cart) back and forth along the tape.

   The program LIGHTDIS records data from each probe every 0.1 seconds for 4 seconds. The data is stored in the CBL unit for later retrieval. It is best to try to move the block apparatus forward, backward, and then forward again within the 4-second period. Keep the cart between 4 feet and 1.5 feet from the light bulb. Roll the cart smoothly and steadily.

   Note: It is important that the height and angle of the two sensors do not change while the cart is in motion. Also, be sure the cart moves in such a manner that:

   a) The two sensors stay along a line perpendicular to the back of the box.

   b) The motion detector's beam is always detecting the back of the box.

   c) The light probe stays directly in front of the light bulb and perpendicular to the back of the box.

4. When the data collection is complete, reconnect the CBL to the calculator with the unit-to-unit link cable. Run the program GETLIGHT on the TI-82 to retrieve the collected data. The program retrieves the data and places it in the following:

   a) Time (measured in seconds) in L2.

   b) Light intensity (measured in milliwatts per square centimeter) in L3.

   c) The distance or separation (measured in feet) between the light bulb and the sensors in L4.

Analysis and Conclusion

1. Until now, no data have been displayed. The purpose of the experiment is to analyze the data that you have collect to determine what function model best represents the relationship between light intensity and an object's distance from a light source. This data set is in L3 and L4. Before this data set is displayed, you should look at a display of *Distance* (separation) vs. *Time* and *Light Intensity* vs. *Time*.

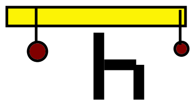
   - Light Intensity vs. Time (L2, L3)
   - Separation vs. Time (L2, L4)
   - Light Intensity vs. Separation (L4, L3)

2. Turn on the STAT PLOT of L2 vs. L4 to display *Distance* vs. *Time*. Press [ZOOM] to select the ZoomStat window. The purpose is not to fit an equation to this data but to look at the pattern formed and compare it to the pattern you see in the plot of L2 vs. L3. Make a printout of this graph using the TI-Graph Link, or a sketch to include in your lab book.

3. Turn off the STAT PLOT of L2 vs. L4 and turn on the STAT PLOT of L2 vs. L3. Press [ZOOM] 9 to select the ZoomStat window. Compare this plot to the printout of the plot of L2 vs. L4.

   What do the plots show about the how the light intensity changes as the distance between the light probe and the light source increases? Discuss the nature of the two data plots and what this implies about the relationship between distance and light intensity.

4. Turn off the STAT PLOT of L2 vs. L3 and turn on the STAT PLOT of L4 vs. L3. Press [ZOOM] 9 to select the ZoomStat window. Discuss the pattern that you see. Find a best-fit equation for the data by going to STAT CALC and computing the most appropriate regression model.



The See Saw Experiment

What relationship exists between the masses suspended and the distances those masses are from the center when the meter stick system is balanced?

Equipment Required

- Meter stick
- String
- Masses

Experiment Procedure

1. Balance the meter stick on the back of a chair (or any other object that can serve as a good fulcrum). It is safe to assume the balance point will be in the middle?

2. Take two short pieces of string, and knot each one so it will fit around the meter stick. Place a string loop on each end of the stick.

3. Place the 200 g mass on one loop at the end of the stick 49 cm from the balance point.

4. Take the 500 g mass, put it in the other string loop, and slide this loop along the stick until the stick is balanced. Record the mass and distance from the balance point on you paper. Also record the point (200, 49) on your paper. This assumes the 200 g mass is 49 cm from the balance point. Do the same operation with the 1000 g mass.

5 Place the 100 g and 200 g masses in the same loop, and move this until it balances. Graph 300 for x and the distance of the loop from the center for the y-value.

6. Graph the four points from above. Guess the equation that best describes the relationship found in the graph. Try to graph the relationship and see if it goes through the four points.

7. Place the 100 g mass at the opposite end of the meter stick from the 200 g that has stayed motionless. Where on the 100 g side of the stick would you have to place the 500 g mass to balance the stick? Check your estimate by actually placing the 500 g mass on the stick?

8. In step 7 you placed a total of 600 grams on one side of the stick. This mass consisted of two weights at two different places, but you could have put the entire 600 g mass at the same place. Use two different methods to find that point.

9. Until now, you have put the 200 g mass at one end of the stick, and the balance point has been at the center of the stick. This time, put the 200 g mass on one end and the 100 gram mass on the other end. Slide the stick until it balances. By this time you have decided that $d_1 m_1 = d_2 m_2$, where

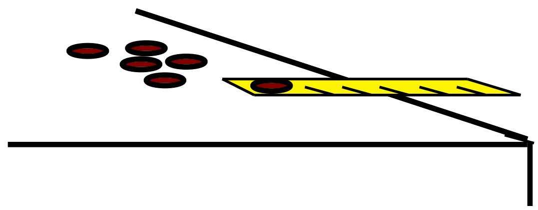
   d_1 = distance the mass is from the balance point on side 1
   m_1 = mass on side 1
   d_2 = distance the mass is from the balance point on side 2
   m_2 = mass on side 2

   Does this relationship hold this time? Why or why not? Use what you have learned to find the mass of the stick.

Penny Functions

- ***You will need:***
- ***centimeter ruler, pencil, and 10 pennies***

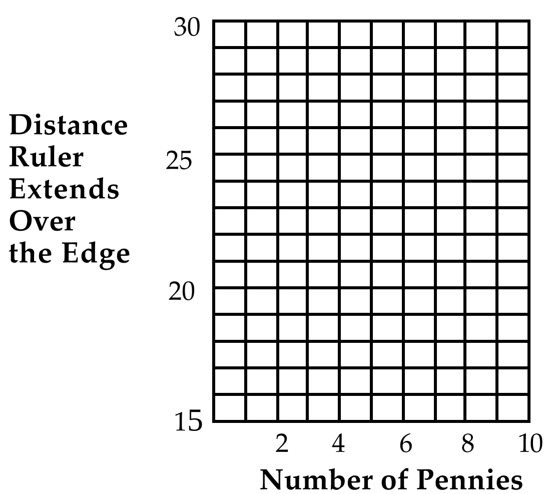
To conduct this experiment, place a ruler on the edge of a desk or table with the end marked zero extending over the edge. Place one penny on the last mark at the other end of the ruler. Use the point of a pencil to push the ruler toward the edge until it starts to tip over. Repeat the experiment using from 2 through 10 pennies.



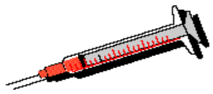
1. Record the data from the experiment in the table below. Let x = the number of pennies on the edge of the ruler. Let f(x) = the distance the ruler extends over the edge (rounded to nearest 0.1 cm.)

x	1	2	3	4	5	6	7	8	9	10
f(x)										

2. Make a graph showing the relationship between x and f(x) in your table. Sketch the curve.



3. Is f(x) increasing or decreasing as x increases?
4. Is the change in f(x) increasing or decreasing?
5. Does f(x) vary directly with x?
6. Does f(x) vary inversely with x?
7. Is the function linear or nonlinear?



Boyle's Law

What relationship exists between the volume of a gas in a container and the pressure it exerts on the container?

Introduction

Before you perform the experiment, try to discover the relationship by thinking about the physical situation you will be performing during the experiment. As you push the hypodermic in, does it get harder to easier to push? What does that tell you about the pressure that is being exerted? As the plunger is being pressed in, what happens to the volume of the gas?

Equipment Required

- CBL
- TI-82 with link
- Vernier Pressure Sensor

Equipment Set-up Procedure

1. Attach the short piece of tubing at the end of the hypodermic to the pressure sensor. The pressure sensor has a three-way valve. Two of the ports seem to be in line, and the third is perpendicular to the other two. The one that is perpendicular is a release valve. Of the other two, one is attached to the pressure sensor itself and the other is attached to the hypodermic.

2. Attach the CBL to the TI-82 and to the pressure sensor.

Experiment Procedure

1. Open the release valve by turning the screw on the top counter-clockwise.

2. Execute the program Pressure. This program can be downloaded from http://www.ti.com.

3. With the release valve still open, hit ENTER when you are asked to zero pressure.

4. Decide how many points you want to enter. (Do at least five.) Enter the number of points, and hit ENTER .

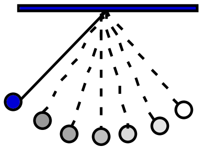
5. Close the release valve, and adjust the plunger to any volume you want. (20 is always a good starting place.) Enter this value. While you hold the plunger at that volume, hit ENTER. A pressure reading will be made. Take care that the plunger is well lined up and doesn't move once ENTER is hit.

6. Repeat step 5 for each of your remaining points. DO NOT OPEN THE RELEASE VALVE for the remainder of the experiment. Be careful to hold the plunger motionless and get it will lined up when measurements are made. Remember, the farther the plunger is pushed in, the harder it will be to hold still.

Analysis and Conclusion

1. Make a scatter plot of your data. What function family do you think best models the pattern of your scatter plot?

2. Use your graphing calculator to determine the mathematical model that best represents your data.



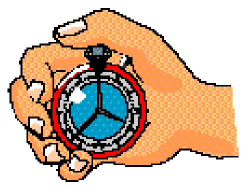
The Pendulum Problem

Introduction

Changing what variable will change the period of the pendulum? What algebraic function represents that relation?

Equipment Required

- string
- masses
- stop watch
- protractor



Experiment Procedure

In order to answer the two questions in the introduction, investigate the following relationships.

- First, by holding the length of the string and the arc constant while suspending different masses, explore the relationship between the period of the pendulum and the mass suspended.

- Second, by holding the mass and the arc constant and varying the length of the string, investigate the relationship between the period and the length of the pendulum.

- Third, by holding the mass and the string lengths constant, explore the relationship between the period and the size of the arc.

Remember that the period of the pendulum is the time it takes for the pendulum to make one complete swing (i.e., the time that it takes for the pendulum to swing over and back to its original position).

Analysis and Conclusion

Use data and graphs to justify your conclusion.

Newton's Law of Cooling

Selected Answers

Answers will vary. These answers are intended only as a sample and were calculated from data using the Fahrenheit scale. The general patterns, however, should be the same.

Experiment Procedure

1. 76 ° F

3. The data given is sample data. If you do not have CBL's, you may wish to use this data, or preferably a subset of this data, enter it in the calculator, and do the analysis portion of this experiment.

L1	L2	L3
0	124.63	------
1	123.89	
2	123.28	
3	122.56	
4	121.59	
5	120.63	
6	119.7	

L1={0,1,2,3,4,5...

L1	L2	L3
7	118.76	
8	117.82	
9	116.55	
10	115.63	
11	114.85	
12	113.95	
13	113.07	

L1(14)=13

L1	L2	L3
14	112.19	
15	111.31	
16	110.66	
17	109.78	
18	108.79	
19	108.28	
20	107.51	

L1(21)=20

L1	L2	L3
21	106.66	
22	106.03	
23	105.39	
24	104.76	
25	104.14	
26	103.32	
27	102.69	

L1(28)=27

L1	L2	L3
27	102.69	
28	102.07	
29	101.46	
30	100.85	
31	100.44	
32	99.824	
33	99.212	

L1(34)=33

L1	L2	L3
34	98.618	
35	98.204	
36	97.61	
37	97.196	
38	96.602	
39	96.206	
40	95.81	

L1(41)=40

L1	L2	L3
41	95.216	
42	94.82	
43	94.424	
44	93.578	
45	93.29	
46	93.038	
47	92.3	

L1(48)=47

L1	L2	L3
48	92.048	
49	91.472	
50	91.076	
51	90.68	
52	90.482	
53	89.942	
54	89.168	

L1(55)=54

L1	L2	L3
55	88.934	
56	89.042	
57	87.998	
58	87.962	
59	87.422	
60	86.882	
61	86.792	

L1(62)=61

L1	L2	L3
62	86.414	
63	86.018	
64	85.64	
65	85.298	
66	85.064	
67	84.668	
68	84.488	

L1(69)=68

L1	L2	L3
69	84.092	
70	83.714	
71	83.516	
72	83.138	
73	82.94	
74	82.562	
75	82.364	

L1(76)=75

L1	L2	L3
76	81.986	
77	81.986	
78	81.248	
79	81.212	
80	81.032	
81	80.312	
82	80.258	

L1(83)=82

L1	L2	L3
83	79.898	
84	79.88	
85	79.52	
86	79.124	
87	79.106	
88	78.944	
89	78.188	

L1(90)=89

L1	L2	L3
90	78.332	
91	78.152	
92	77.792	
93	77.594	
94	77.756	
95	77.036	
96	76.82	

L1(97)=96

L1	L2	L3
92	77.792	
93	77.594	
94	77.756	
95	77.036	
96	76.82	
97	76.64	
98	76.64	

L1(99)=98

4. Your graph should look similar to the one shown in Figure 1. Please note that this is a scatter plot. The data points are so close together that the plot looks as though it is continuous.

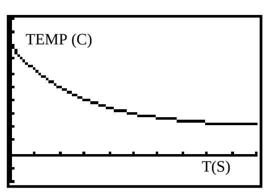
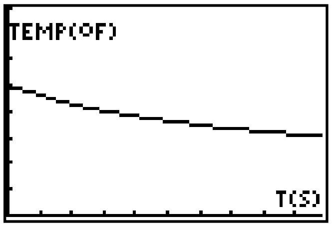
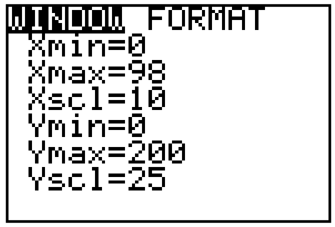


Figure 1

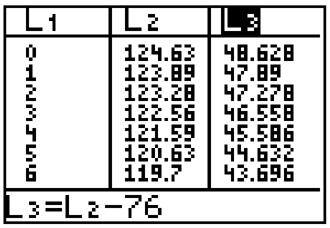
Analysis and Conclusion

1.





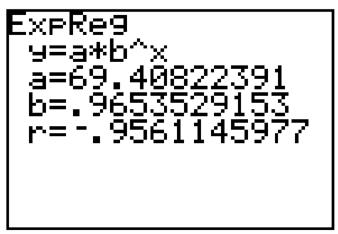
2.

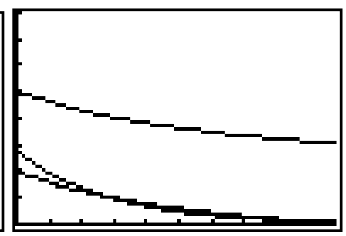


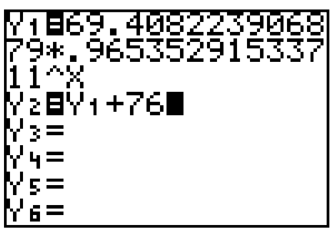
L1	L2	L3
0	124.63	48.628
1	123.89	47.89
2	123.28	47.278
3	122.56	46.558
4	121.59	45.586
5	120.63	44.632
6	119.7	43.696

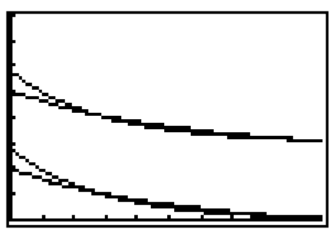
L3=L2-76

3.







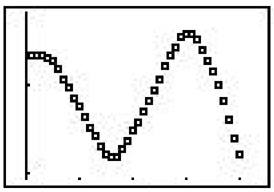


Light Intensity

Selected Answers

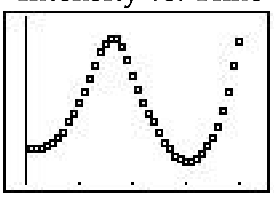
Analysis and Conclusion

2. **Distance vs. Time**

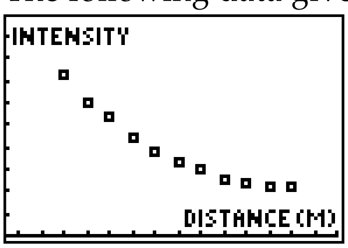


Students should compare these two plots to see that as the separation distance decreases, the intensity of the light increases, and vice versa.

3. **Intensity vs. Time**



1 The following data gives separation distance in L1 and intensity in L2.



L1	L2	L3
1	.18147	------
1.1	.14994	
1.2	.13547	
1.3	.11211	
1.4	.09282	
1.5	.08206	
1.6	.0739	

L1={1,1.1,1.2,1...

L1	L2	L3
1.5	.08206	
1.6	.0739	
1.7	.06352	
1.8	.05981	
1.9	.05498	
2	.05498	

L1(12)=

Using the data given above and the power regression model on the graphing calculator, you find $y = \frac{0.18}{x^{1.84}}$. This is close to a common model given in mathematics textbooks as an example of inverse variation:

$$I = \frac{k}{d^2}$$

where I = light intensity, d = distance between the light source and the object measuring intensity, and k = a constant.

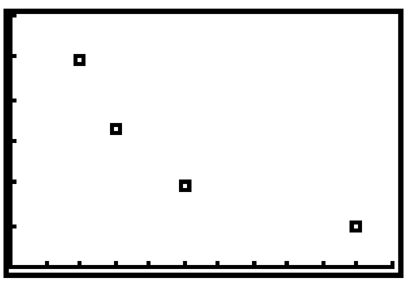
The See Saw Experiment

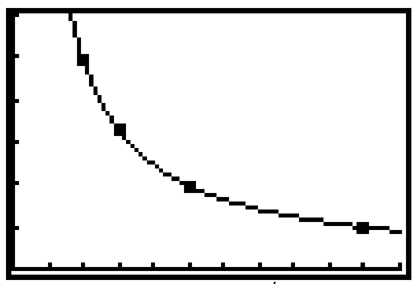
Selected Answers

1–5.

Mass (g)	Distance (cm)
200	49
500	19.5
1000	9.9
300	33

6.





y = 9800/*x*

7. 9.8 cm from the balance point.

8. 16.3 cm from the balance point. You can determine this by using 600 g as the mass on one side, or you can determine the balance point for the half of the meter stick with the 100 g and 500 g masses.

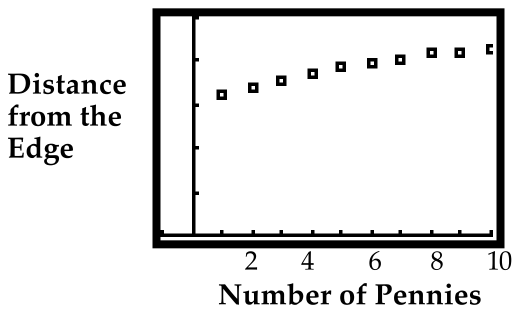
Penny Functions

Selected Answers

1.

x	1	2	3	4	5	6	7	8	9	10
f(x)	*16.0*	*17.0*	*17.8*	*18.4*	*19.2*	*19.7*	*20.1*	*20.6*	*21.0*	*21.4*

2.



3. *f*(x) is increasing at a decreasing rate.

4. Decreasing.

5. If f(x) to varies directly with x, then $f(x) = kx$. There are two problems. First, the graph is not a straight line; and second, the graph does not appear to pass through the origin.

6. No, f(x) does not seem to vary inversely with x. If this were inverse variation then $f(x) = k/x$. In this case the x and y axes would be asymptotes, which is not the case.

7. The function is nonlinear because the rate of change is not constant.

Extension: The model for this experiment cannot be found from the calculator regressions. This is a case where the students have to think about the physics of the lever and derive the model algebraically. The relates back to what they learned in *The See Saw Experiment.*

The model for this particular data set is $y = \dfrac{77.43x + 483}{2.67 + 32.2}$. The ruler is 30 cm long, and has a mass of 32.2 grams. The average penny mass is 2.67 grams. The asymptote for this curve is of particular interest.

$y = \dfrac{77.43x + 483}{2.67 + 32.2} = \dfrac{77.43 + \dfrac{483}{x}}{2.67 + \dfrac{32.2}{x}}$ which means y = 29 as $x \to \infty$.

The asymptote of 29 indicates that the farthest the ruler can hang over the edge is 29 cm. This is what you would hope to find, since the length of the ruler is 30 cm and the center of mass of the pennies is 1 cm from the end.

The general equation is $y = \dfrac{M_r \dfrac{L}{2} + M_p x(L - y - z)}{M_r + M_p x}$ where

$x =$ number of pennies

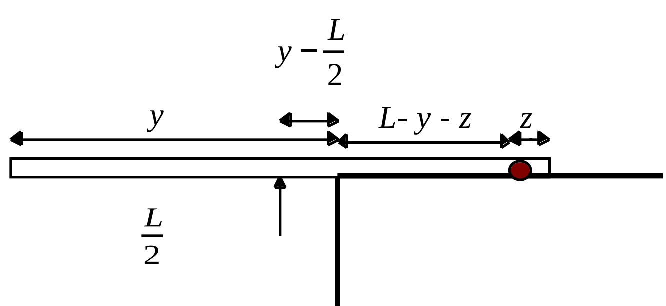
$y =$ overhang distance

$M_r =$ 32.2 g, mass of the ruler

$M_p =$ 2.67 g, mass of average penny

$L =$ 30 cm, length of ruler in cm

$z =$ 1 cm, distance penny is from end of ruler



For the system to be in balance: $M_r(y - \frac{L}{2}) = xM_p(L - y - z)$. When this is simplified it is equivalent to the equation given above. By using what they know about levers, algebra, and the specific values for this experiment students can derive the algebraic model.

$$32.2(y - 15) = 2.67x(29 - y)$$

$$32.2y - 483 = 77.43x - 2.67xy$$

$$32.2y + 2.67xy = 77.43x + 483$$

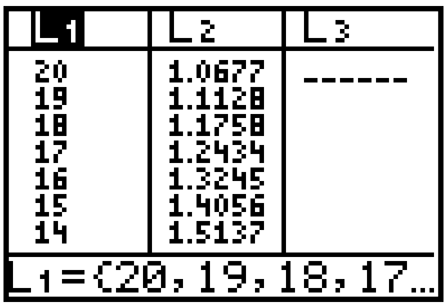
$$y(32.2 + 2.67x) = 74.76x + 483$$

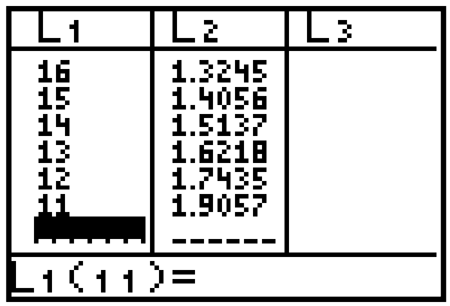
$$y = \frac{77.43x + 483}{2.67x + 32.2}$$

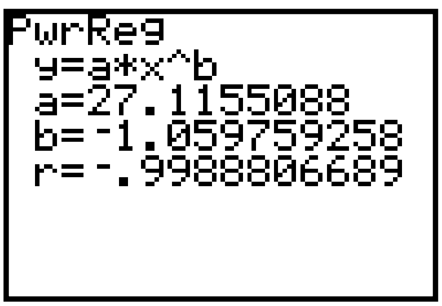
Boyle's Law

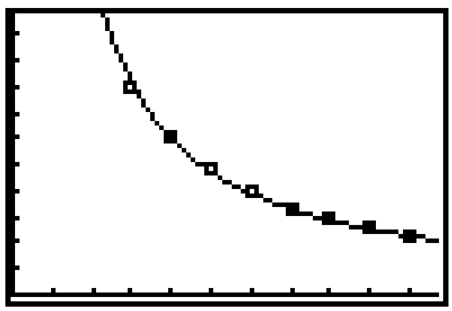
Selected Answers

Data:









As the pressure decreases, the volume increases. This suggests an inverse variation.

The Pendulum Problem
Selected Answers

Changing what variable will change the period of the pendulum?

Students should find that the period of the pendulum is determined by the length of the string. The other variables do not effect the period.

What algebraic function represents that relation?

$$P = 2\pi\sqrt{\frac{L}{g}}$$ where :

P is the period
L is the length of the string
g is the force of gravity

Students will use the power regression model on the graphing calculator to get an equation something like $P = kL^{\frac{1}{2}}$.