



The Yo-Yo Problem

(Solving Linear Equations)

Objective

Students explore linear patterns, write a pattern in symbolic form, and solve linear equations using algebra tiles, symbolic manipulation, and the graphing calculator.

Overview of the Lesson

The lesson starts with the presentation of the yo-yo problem. Students then complete a hands-on activity involving a design created with pennies that allows them to explore a linear pattern and express that pattern in symbolic form. Algebra tiles are introduced as the students practice solving linear equations. Working from the concrete to the abstract is especially important for students who have difficulty with mathematics, and algebra tiles help students make this transition. In addition to using algebra tiles, students also use symbolic manipulation and the graphing calculator. Finally, the students return to solve the yo-yo problem. A very special feature of this lesson is the effective use of peer tutors in this inclusion classroom.

Materials

- graphing calculator and overhead unit
- overhead projector
- overhead algebra tiles

For each group:

- 31 pennies
- set of algebra tiles
- graphing calculators

Procedure

1. **Introduction of the Problem:** Explain the details of *The Yo-Yo Problem* to the class.

The Yo-Yo Problem

Andy wants to buy a very special yo-yo. He is hoping to be able to save enough money to buy it in time to take a class in which he will learn how to do many fancy tricks. The 5-ounce aluminum yo-yo costs \$89.99 plus 6% sales tax. Andy has already saved \$17.25, and he is earning \$7.20 a week by doing odd jobs and chores. How many weeks will it take him to save enough money for the yo-yo?

Have the students calculate the total amount of money he will have to save by determining the sales tax and adding that amount to the price of the yo-yo. Before students begin to solve the problem, you should review linear patterns and have students practice solving linear equations.

2. **Penny Pattern Exploration:** For the second part of the lesson, students create a design in stages. The first stage is one penny surrounded by six pennies. For each successive stage, six more pennies are added to the outside of the pattern. Have students continue to make several more stages of this design with their groups. They should create a table of values using n for the stage number and p for the number of pennies used. Finally, have each group determine an algebraic rule representing the relationship between the stage number and the number of pennies used. Then have each group share its rule with the entire class.
3. **Solving Equations Using Algebra Tiles:** After distributing the algebra tiles, write the equation to be solved, $4x + 3 = 3x - 1$, on the board. Demonstrate the use of the algebra tiles while the students work the problem with their groups. Ask questions to help draw out the strategies for solving the equation. Students should remember that the goal is to isolate the variable on one side of the equation. Students accomplish this by performing the same operations on both sides of the equation. Have students check their solutions using symbol manipulation.

The second problem to give the students to solve, $3(x + 1) - 1 = 2(2x - 3)$, involves the distributive property. Have one pair of students solve the problem at the overhead using the algebra tiles, have another pair of students work at the board using symbol manipulation, and have the rest of the class work in groups at their seats. In each group working at their seats, make sure that some students use algebra tiles, and have the others use symbol manipulation.

- Solving Equations Using the Graphing Calculator:** After the students have completed solving the equation, $3(x + 1) - 1 = 2(2x - 3)$, using algebra tiles and symbol manipulation, model solving the equation using the graphing calculator. Have a student, along with a peer tutor, come to the overhead and follow your directions to solve the equation. Enter the left-hand side of the equation into Y1 and the right-hand side into Y2. After students have graphed both equations, have them look to see where the lines intersect. You should adjust the viewing so that the intersection of the two lines is visible. After changing the viewing window, students can use the trace key to find the intersection point, and they should also examine the values in the table to determine what the x -value is when Y1 and Y2 are the same. The students should see that the function values are the same when $x = 8$. This answer agrees with the solutions determined using the algebra tiles and symbol manipulation.
- Back to The Yo-Yo Problem:** Review the basic facts of *The Yo-Yo Problem* for the students and direct them to work with their groups to solve the problem. Have students use various methods to determine the solution, including writing a symbolic equation and solving it, using the trial and error method, and using simple arithmetic. Have students go the board and present their solutions to the class.

Assessment

This lesson offers many opportunities for ongoing assessment. As students work in groups and as they make presentations to the class, you can evaluate their mathematical understanding. This lesson also gives students many opportunities to connect ideas from the various activities and use those ideas as they work to solve *The Yo-Yo Problem*. Did they see *The Yo-Yo Problem* as another linear pattern? What was the initial value, and what was the rate of change? Could they express this symbolically? Could they solve the symbolic representation? Do they have generally good problem solving ideas?

Teachers have the very difficult task of analyzing such situations and using their observations to shape future activities for students. Any one lesson is just a small part of the mathematical journey on which students are traveling. What is the destination? Teachers need to have a clear vision of what the goal is. They need to have a firm understanding of what it is they are trying to achieve in order to be able to effectively assess whether they are successful.

Extensions & Adaptations

- You could have students write problems that are similar to *The Yo-Yo Problem*. They could share their problems with the entire class. You could assign some of these problems as class work or homework. You could also display the problems on a bulletin board in the classroom.

- Have students bring in examples of linear models. Ask them to explain why the relationship is a linear model. Also have the students bring in examples that are not linear and explain why they are not linear.
- As a follow-up activity, ask the students to write an equation for the relationship between the pattern number, n , and the number of pennies required to make the pattern. Also, have them draw the graph and make a table of values. Have them do the same thing for the relationship between the number of weeks, w , and the amount of money saved in *The Yo-Yo Problem*. Then ask the students to compare the equations, graphs, and tables and describe how they are different and how they are the same.

Mathematically Speaking

Linear functions have been and will continue to be a fundamental part of the study of algebra as mathematics education develops to meet the needs of the ever-changing technological world of today. These functions help us to better understand the world around us, and to answer problems and explain phenomena from many diverse fields, including physics, biology, and economics.

In the traditional approach to algebra, students generally learn how to solve linear equations before they study linear models. Traditionally, solving equations was devoid of any real-world context until after the symbol manipulations were mastered. However, some of the newer materials emphasize understanding the mathematical model as it is related to a real-world application, and then building on that conceptual understanding in order to help students solve equations.

In this lesson, students work with linear situations involving a penny pattern and the total amount of money saved over a period of time. Helping students understand the basic patterns involved in linear models—in their symbolic form, graph form, and table form—gives them a firm foundation for solving problems involving linear situations. It also gives students more ways to solve these types of problems. They are not limited to using symbol manipulation. They can solve an equation using a graph or a table of values in addition to using symbol manipulation.

One of the major goals of mathematics education is developing good problem-solving skills. Students need a solid understanding of multiple representations of various functions, including linear functions, in order to be considered mathematically literate. Teaching from this perspective, and making multiple opportunities available to help students make connections and develop these skills is very important in today's mathematics classroom.

Tips From Ellen

The Inclusive Classroom

Although the model may vary from classroom to classroom, school to school, and system to system, inclusive education is a goal and a reality for all educators. Inclusion is defined as a process in which a student with disabilities is placed as a full participating member of a general education class, to the extent appropriate, in the same school as his or her non-disabled peers. Supplemental aides and services are provided, as necessary, to support the learning of individual and relevant objectives.

Where it is necessary to modify the curriculum, the delivery of instruction, or method of student performance to meet the special needs of students, the approaches most likely to occur in algebra classes or higher are as *accommodation* and *adaptation*.

Accommodation: A modification that does not significantly change the content or conceptual difficulty of the material (example: having students complete every second problem on an assignment; having students dictate answers).

Adaptation: A modification that changes the content or conceptual difficulty of the material (examples: allowing students to use a calculator to complete an assignment; allowing memory aids).

Fortunately for educators, the move toward full inclusion parallels and incorporates what we are learning about best practice for instruction for all students. Among the following general adaptations designed to promote the success of special needs students, teachers are sure to find common sense suggestions for teaching in general and teaching reform mathematics in particular:

- ✓ Establish a climate that welcomes all students and promotes the ABC's — acceptance, belonging, and community.
- ✓ Use techniques such as cooperative learning to encourage class participation.
- ✓ Use a multisensory approach to teaching and learning.
- ✓ Sequence instruction to include the concrete (manipulative), semi-concrete (pictorial representation), and symbolic levels.
- ✓ Model what is expected.
- ✓ Value multiple approaches to problem solving.
- ✓ Encourage students to be aware of their own learning style preferences and to be responsible for their own adaptations where possible.
- ✓ Be clear about objectives; provide advance organizers for lessons or units of instruction.

- ✓ Give a few directions at a time and check for understanding.
- ✓ Establish an area on the board to record daily assignments.
- ✓ Encourage students' interests and talents by allowing projects instead of written assignments.
- ✓ Frequently monitor all students' comprehension (without drawing specific attention to those with special needs).
- ✓ Provide consistent, specific feedback; try to reinforce the positive rather than draw attention to the negative.
- ✓ Provide students opportunities to correct errors as part of the assessment process.
- ✓ Provide clearly typed worksheets and tests; use a consistent format in worksheets and tests.
- ✓ Vary the types of responses on tests (true/false, multiple choice, short answer) and have each type in a separate section.
- ✓ Provide extra time on assignments or tests as needed.
- ✓ Allow and encourage the use of a tape recorder in class or on review days.
- ✓ Teach and encourage the use of study aids, such as underlining of key words, mnemonic devices, graphic organizers, and note-taking.
- ✓ Encourage students to assess their own progress.

Of course, there are many more specific modifications and adaptations that you might make, depending upon the special needs of your students. These are best determined through collaboration with your special education resource teachers and the students themselves, but many teachers find that good teaching in inclusive classrooms is simply good teaching.

Resources

McConnell, John et al. *The University of Chicago School Mathematics Project, Algebra, Second Edition*. Scott Foresman. Glenview, IL: 1996.

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics Addenda Series, Grades 9 - 12 : Algebra in a Technological World*. Reston, Virginia: National Council of Teachers of Mathematics, 1991.

Internet location: <http://www.math.utep.edu/sosmath/algebra/linearequations.mathhome.htm>

This is part of S.O.S. Math, which is a site providing tutorials in a wide variety of mathematical topics. This location takes you directly to a tutorial for solving linear equations. It includes six problems to view with a step-by-step tutorial on how to solve each one. It also has practice problems with answers. From this location you can go to the homepage for S.O.S. Math, whose address is <http://www.math.utep.edu/sosmath/algebra/algebra.html>

Internet location: <http://www.iit.edu/~smile/ma9525.html>

This is a delightful lesson on solving linear equations. This site has hundreds of lessons. The lessons are divided by topic, but unfortunately, not by grade level. It is a very worthwhile site, and you will find all sorts of lessons.

Ideas for Online Discussion

(Some ideas may apply to more than one standard of the NCTM Professional Standards for Teaching Mathematics.)

Standard 1: Worthwhile Mathematical Tasks

1. How do real-world applications help make mathematics meaningful? Where do you find problems that are practical applications of meaningful mathematics?
2. Which different methods did the students in this class use to solve linear equations? Which methods do your students use most often? How can a teacher encourage multiple approaches to solving problems?

Standard 2: The Teacher's Role in Discourse

3. How does a teacher balance the need to expand and clarify a lesson, and also keep within the time constraints of the class period?

Standard 3: Students' Role in Discourse

4. In this video lesson, the teacher mentioned that she always had the students come to the board or the overhead in pairs. Did this seem to help the students with their presentations? What methods do you use to help students feel more comfortable in making presentations to the class?

Standard 4: Tools for Enhancing Discourse

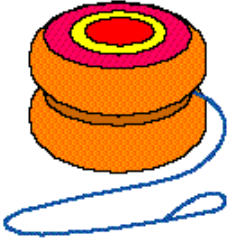
5. *The Penny Pattern* activity could have been done by simply drawing the patterns rather than actually physically handling the pennies. Discuss the pros and cons of actually using pennies.
6. Do you use algebra tiles with your students? Discuss the benefits of this hands-on manipulative. Are there any activities that work especially well for you?

Standard 5: Learning Environment

7. In this class, the peer tutors served in an important capacity. How did they affect the learning environment and how did they aid the student discussions?

Standard 6: Analysis of Teaching and Learning

8. Consider the activities completed during this lesson. What assessment do you think the teacher was making? What types of activities would you plan for this class as a follow-up to this lesson?



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Part I

1. How much sales tax will Andy have to pay?
2. What will be the total cost of the yo-yo, including tax?

Part II

3. Let w be the number of weeks that it will take Andy to save enough money to buy the yo-yo. Write an algebraic equation that will help you solve the problem.
4. Solve your equation for w , and check your answer. Be prepared to present your solution to the class.



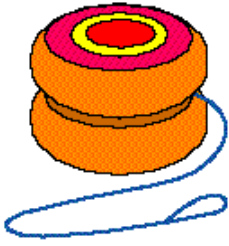
The Penny Pattern

Exploring Linear Models

1. Create a pattern using pennies. Stage one of the pattern is shown next to the title above—one penny surrounded by six additional pennies. To create each additional stage of the design, place more pennies extending out from the six that surround the center penny. Continue making this design until you have used up all of your pennies. On the back of this sheet, sketch the first four stages of the pattern.
2. Using your penny pattern or the sketches of your penny pattern, create a table of values.

Stage Number, n	1	2	3	4	5
Number of Pennies Required, p					

3. How many pennies are needed to make stage 6, stage 7, and stage 8 of the penny pattern? How did you determine your answer?
4. Write an algebraic model that expresses the relationship between the stage number, n , and the number of pennies required to make that design, p .
5. Use your model to determine how many pennies are needed to make stage 80, stage 95, and stage 100 of the penny pattern.
6. Using your graphing calculator, make a scatter plot of the table of values from problem 2. Graph your model from problem 4 to determine if it is correct, and then use the graphing calculator to create a table of values to check your answers to problems 3 and 5.
7. If you use 127 pennies to make the penny pattern, how many pennies will be in each spoke coming out from the center penny? Can you find this answer three different ways?



The Yo-Yo Problem

Selected Answers

Part I

1. \$5.40
2. \$95.39

Part II

3. $17.25 + 7.20w = 95.39$
4. $w = 10.8528$ or approximately 11 weeks.



The Penny Pattern

Exploring Linear Models

Selected Answers

2.

Stage Number, n	1	2	3	4	5
Number of Pennies Required, p	7	13	19	25	31
3. Stages 6, 7, 8 require 37, 43, and 49 pennies.

4. $p = 1 + 6n$
5. Stages 80, 95, and 100 require 481, 571, and 6001
7. Solving $1 + 6n = 127$ for n gives 21.