

Roll Out The Barrel

(Computation and Estimation)

Objective

Students will apply **mathematical modeling** to solve a real-world problem.

Overview of the Lesson

Students are presented with a problem in which their manufacturing company is given two options for storing oil barrels. These options include a square storage unit and a rectangular storage unit, (twice the area of the square unit). Their task is to decide which of the storage units will provide the most inexpensive space to store 125 containers. Students are provided 35mm film canisters and paper mats to model storage arrangements for each unit. Students find that the arrangement of the cylinders in the storage unit will affect the cost. Students work in small groups collecting the data, performing the necessary computations, and constructing reasons for their unit selection.

Materials

Each Group

- ❶ Two square paper mats ($8\frac{1}{4}$ X $8\frac{1}{4}$ inches)
- ❷ Approximately 65 empty film canisters (Film canisters can usually be obtained at no cost from film developing centers.)
- ❸ Calculator
- ❹ Data Collection Worksheet

Procedure

Begin the lesson by asking students to assist you in solving a storage dilemma:

There are 125 oil barrels which need to be stored for two months. There are two storage options available. Given these two options, the task is to find the most economical one for storing these barrels.

Option 1: The barrels can be stored in square storage units at a cost of \$67 per month. The height of this unit allows for the barrels to be stacked two-high.

Option 2: The barrels can be stored in rectangular storage units, twice as long as the square storage unit, at a cost of \$105 per month. The height of this unit allows for the barrels to be stacked two-high.

Note 1: In the video, the teacher presents the information in stages. This allows for very careful monitoring of the students' level of understanding. In addition, students remain cognizant of the information that they know and that more information is needed to solve the problem. The teacher provides more and more information as the students request it.

Note 2 The dimensions of the scaled square storage unit will depend on the size of the cylindrical manipulative used to represent the barrels. If 35 mm film canisters are used, and the canisters are placed top side down, the mat dimensions are $8\frac{1}{4} \times 8\frac{1}{4}$ inches. A copy of this mat is provided with this guide. If some other materials are used to represent the barrels, scale the mat so that the length and width both measure $5\frac{1}{2}$ barrels. When the floor mat for the rectangular unit is constructed, the length should measure 11 barrels and the width should still measure $5\frac{1}{2}$ barrels. Thus, the rectangular floor mat can be constructed by placing two square mats together. Using the film canisters this measurement is $16\frac{1}{2} \times 6\frac{1}{4}$ inches.

Place students in small groups. Inform them that they will be modeling this problem by using film canisters to represent the barrels and paper mats to represent the area of the floor of the storage units. Have them first experiment to determine the maximum number of barrels that will be able to fit in the square unit. Once they have decided on an arrangement and the number of corresponding barrels, have them record this information on their Data Collection worksheet.

Reassemble the class to discuss the findings. Generally students discover one of two arrangements — a regular pattern or a staggered pattern. The regular pattern illustrates 5 rows of barrels with 5 barrels in each row, resulting in 25 barrels per layer. Therefore the number of barrels that can be placed in the square storage unit is 50. Another solution that students discover is one that results from arranging the barrels in a staggered pattern. In this case, students are able to arrange the barrels in

6 rows with 5 barrels in each row. Thus, there are 30 barrels in layer one and 30 barrels in layer two, totaling 60 barrels per square storage unit.

Based on the information obtained from arranging the barrels in the square storage unit, have students estimate the number of barrels that will be able to fit in the rectangular storage unit. The estimate will probably be twice as many, since the room is twice as large.

Next have students discover the exact number of barrels that will fit in the rectangular storage unit by proceeding in a manner similar to the one above. It should become clear that in the regular pattern, 11 barrels can be placed in 5 rows, resulting in 55 barrels per layer. In the staggered pattern, 6 rows of barrels can be placed. However, 3 of these rows will contain 10 barrels and 3 of the rows will contain 11 barrels. The total number of barrels in one layer will therefore be $(3 \times 10) + (3 \times 11)$, which is 63.

Revisit the estimate made earlier, and discuss why **more than** twice as many barrels were able to fit into the unit which was exactly twice as large.

To continue, inform students that the square rooms are rented at the rate of \$67 per month and the rectangular rooms are rented at the rate of \$105 per month. Based on the number of barrels which need to be stored, which arrangement and which storage unit would be the most economical one to select?

Students should again work in their groups to arrive at a solution for this problem. Ultimately they should discover that:

- It would take 3 square storage units to store the 125 barrels regardless of their arrangement. The cost would be \$201.
- If the regular pattern is used to arrange barrels, then two rectangular storage units would be required and the cost would be \$210.
- If the barrels are arranged in the staggered pattern and stored in the rectangular storage unit, only 1 unit would be needed and the cost would be \$105.

Mathematically Speaking...

Mathematical modeling is a type of problem solving. Frequently in a mathematical modeling situation, a phenomenon that is seemingly non mathematical in context must be modeled. This may be an event in the realm of politics, such as predicting election result; of economics, such as finding the long term behavior of oil prices; or even ecology, such as predicting the future growth pattern of a forest.

Important factors must be discerned, relationships must be determined, and these relationships must be mathematically interpreted. The mathematical

interpretations of relationships allow for an analysis of the phenomenon so that conclusions can be found. This mathematical modeling is a systematic process that draws on many skills and employs the higher cognitive activities of interpretation, analysis, and synthesis.

Extensions & Connections

Packing problems are always lots of fun for students. For example, suppose instead of 125 barrels, there were 225 barrels to be stored. There were three options, the square unit, the rectangular unit (twice as long as the square unit), and a rectangular unit $1\frac{1}{2}$ times as long as the square unit. This new unit rents for \$80 per month. What would be the best storage option?

There are many things in our everyday lives which are packed. In some cases, the packaging cost more than the product itself. Have students investigate how much it costs to pack items such as pizzas, sodas, or other goods. Students could construct questions, conduct interviews of local businesses, and give a report on their findings. Questions might explore such topics as: Why are they packed in certain quantities rather than other quantities? Are there any products that are packed in a staggered pattern?

Resources

National Council of Teachers of Mathematics. *Mathematical Modeling in the Secondary School Curriculum*. (1991) Reston, Virginia.

Ideas for Online Discussion

*(Some ideas may apply to more than one standard of the **NCTM Professional Standards for Teaching Mathematics.**)*

Standard 1: Worthwhile Mathematical Task

- ① As stated in the lesson, mathematical modeling problems usually are an outgrowth of a real-world phenomenon. Share a problem that students would solve using mathematical modeling.

Standard 2: Teacher's Role in Discourse

- ② Discuss one of the techniques the video teacher utilized to encourage discourse.

Standard 4: Tools for Enhancing Discourse

- ③ This lesson employed everyday items as a math manipulative. What items have you used to teach mathematical principles and how well did it serve its purpose?

Standard 6: Analysis of Teaching and Learning

- ④ The key to the success of any lesson is careful planning and preparation. Share one of your secrets for effective and efficient planning and preparation.

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Data Collection Worksheet

Square Storage Unit

Type of Pattern	Number of Rows	Number per Row	Total Number with One Layer	Total Number with Two Layers	Cost per Room	Total Number of Rooms Needed	Total Cost

Rectangular Storage Unit

Type of Pattern	Number of Rows	Number per Row	Total Number with One Layer	Total Number with Two Layers	Cost per Room	Total Number of Rooms Needed	Total Cost