PBS Mathline®

The Middle School Math Project

Let's Face It

(Geometry)

Objective

Students will be able to identify the five regular polyhedra (tetrahedron, hexahedron, octahedron, dodecahedron and icosahedron) and describe basic characteristics about each.

Overview of the Lesson

Students are introduced to the idea of three-dimensional figures by first reviewing polygons and regular tessellations. Comparisons are made between regular tessellations and five special polyhedra. These polyhedra, known as the Platonic Solids are the tetrahedron, hexahedron, octahedron, dodecahedron and the icosahedron. Students are provided with the necessary materials to construct these shapes. However, they must decide how to build them on their own. Using nets as blueprints, students construct the polyhedra and record the number of faces, edges and vertices for each. Finally, they are asked to look for a relationship between the number of faces, edges and vertices, (Euler's formula).

Materials

Each group:

❶ 1 bag of rubber bands

❷ 1 roll of scotch tape

❸ 24 flexible drinking straws (per person)

❹ 2 calculators

❺ String

Materials *(continued)*

❻ Copies of nets: Tetrahedron, Hexahedron, Octahedron, Dodecahedron, Icosahedron

❼ Student Worksheet: Polyhedra

❽ Cardboard geometric shapes *(optional)*

Procedure

Introduce the concept of polyhedra by building on students' previous experiences with regular polygons and regular tessellations. Center the discussion around reviewing the definition of a regular tessellation and how this definition connects to polyhedra in general and the platonic solids in particular. Point out that there are five Platonic Solids. They are the tetrahedron, hexahedron, octahedron, dodecahedron and the icosahedron. ***Note:*** *See Mathematically Speaking . . .*

Provide students with the Polyhedra Worksheet containing: the names of the five polyhedra, the number and name of the regular polygons which are the faces of each polyhedra, and a chart for recording the number of faces, vertices and edges. This worksheet provides a framework for the students to begin their explorations and dialogue. Also, provide students with copies of nets (blue prints) for all five polyhedra.

Students should work together in small groups to construct and investigate the tetrahedron and hexahedron. In the video, students have had previous experiences with the hexahedron (prism and cube), and the tetrahedron (triangular pyramid) However, if this is the first formal introduction of these geometric shapes with your students, you will definitely want them to construct the tetrahedron and the hexahedron. Following construction, they should record the number of faces, vertices and edges on the Polyhedra Worksheet.

If possible, each student should construct an octahedron from straws. This will allow each student to have a personal model to take home to share with parents. It may also inspire them to build the other two polyhedra from straws at home during their spare time. (Even though students are building their own model, allow them to assist each other with the constructions.) Students should discuss their findings and record them in the appropriate column.

Note*: Take the time to ensure that students know how to assemble the polyhedra with the rubber bands and the cardboard pieces and also how to construct the faces and connect the faces using the straws and the scotch tape.*

Once all of the polyhedra have been constructed and the information recorded, students should study the entries in the chart to determine if there is a relationship between the number of faces, vertices and edges. Depending on the level of the students, decide on an appropriate amount of assistance to provide in guiding them to the discovery of Euler's Formula: $F + V = E + 2$. Finally, discuss where these polyhedra exist in the real world.

Mathematically Speaking. . .

A tessellation is an arrangement of closed shapes that completely covers the plane without overlapping and without leaving gaps. When a tessellation uses only one shape, it is called a pure tessellation. The video teacher refers to a regular tessellation as a pure tessellation which is created from using only regular polygons. A polyhedron is a solid formed by flat surfaces enclosed by polygons. The flat polygonal surfaces of a polyhedron are called its faces. An edge of a polyhedron is a segment where two faces intersect. A vertex of a polyhedron is a point of intersection of three or more edges.

There are only five regular polyhedra. This means that there are only five solids in which all of the faces are congruent regular polygons. These five regular polyhedra are called the Platonic Solids. The Platonic Solids are: the tetrahedron which has 4 equilateral triangles as faces; the hexahedron which has 6 squares as faces; the octahedron which has 8 equilateral triangles as faces; the dodecahedron which has 12 equilateral pentagons as faces; and the icosahedron which has 20 triangles as faces.

Polyhedra	No. of Faces	No. of Vertices	No. of Edges
Tetrahedron	4	4	6
Hexahedron	6	8	12
Octahedron	8	6	12
Dodecahedron	12	20	30
Icosahedron	20	12	30

Extensions & Connections

There is an interesting history surrounding the Platonic Solids. Have students research how they were discovered and some of the meanings attached to them.

Have students investigate the role that the Platonic Solids play in our everyday lives.

Students may want to experiment to see if they can construct other solid figures using two or three different regular polygons.

Resources

National Council of Teachers of Mathematics. Curriculum and Evaluation Standards for School Mathematics Addenda Series: Geometry in the Middle Grades. (1992) Reston, Virginia.

Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

Standard 1: Worthwhile Mathematical Tasks

❶ Often, activities are selected based on the anticipated amount of time it will take to complete them rather than on the quality of the activity itself. For example, the construction of the polyhedra is a very time consuming task. Share your thoughts on the importance of task verses time.

Standard 2: Teacher's Role in Discourse

❷ When concrete materials are utilized, students are usually excited and discussion begins spontaneously and sometimes get off track. How do you encourage them and yet, get them back "on track?"

Standard 4: Tools for Enhancing Discourse

❸ The activities in this lesson build abstract concepts using manipulatives. How can you ensure that students understand and internalize abstract objectives when using manipulatives?

Standard 5: Learning Environment

❹ Describe how you adjust the classroom environment when your students are assigned tasks such as making models.

Standard 6: Analysis of Teaching and Learning

❺ How do you assess individual student learning when they are engaged in concrete activities and are assimilating the information with other members of their group?

❻ What is your role when all of the students in your class are actively involved in the assigned tasks and appear not to need assistance from you?

Polyhedra

Name: _____

There are five regular polyhedra. A regular polyhedron has regular polygons for faces and the same number of faces at each vertex.

Using the materials provided, make the five regular polyhedra. You'll need the following:

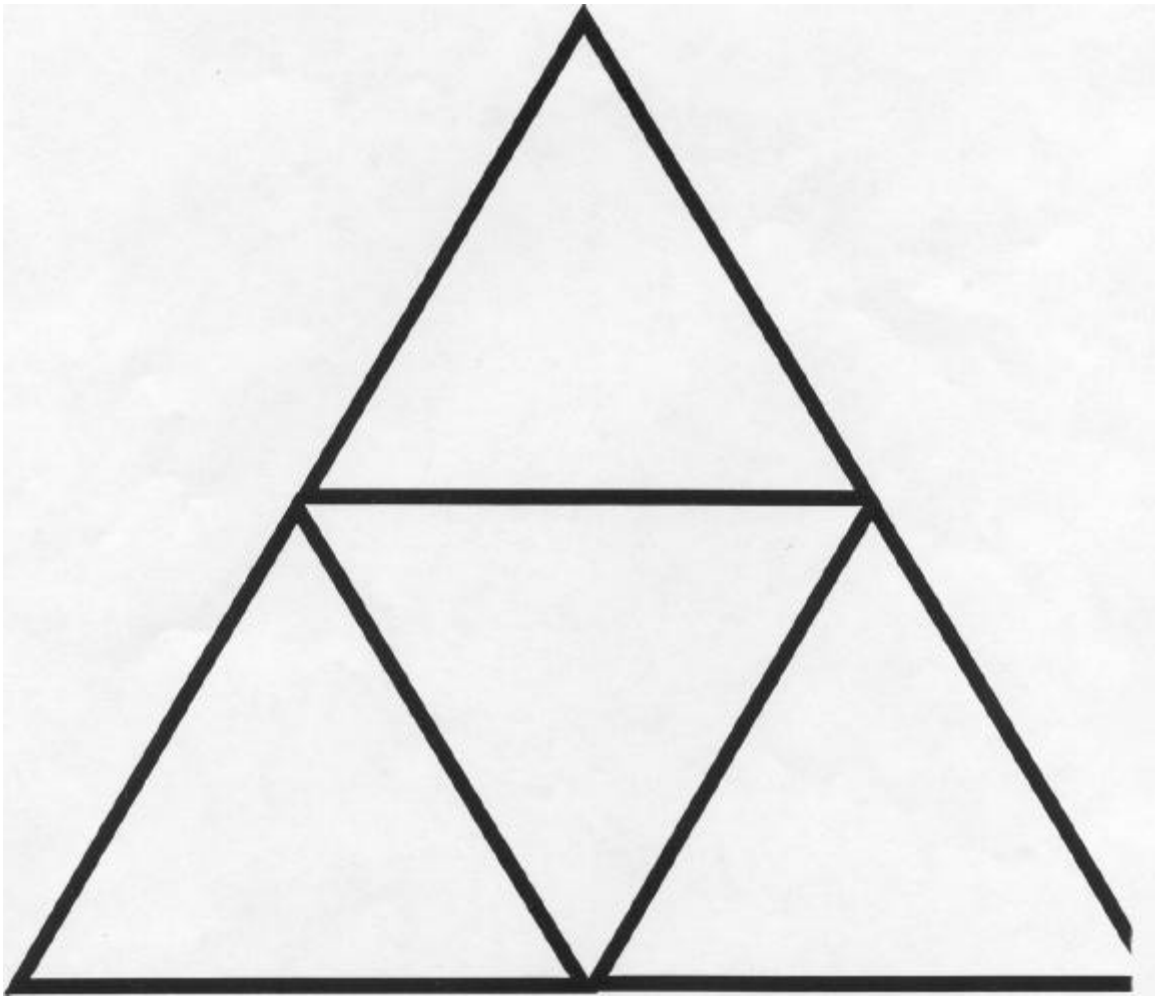
- 4 equilateral triangles for a tetrahedron
- 6 squares for a hexahedron
- 8 equilateral triangles for an octahedron
- 12 regular pentagons for a dodecahedron
- 20 equilateral triangles for an icosahedron

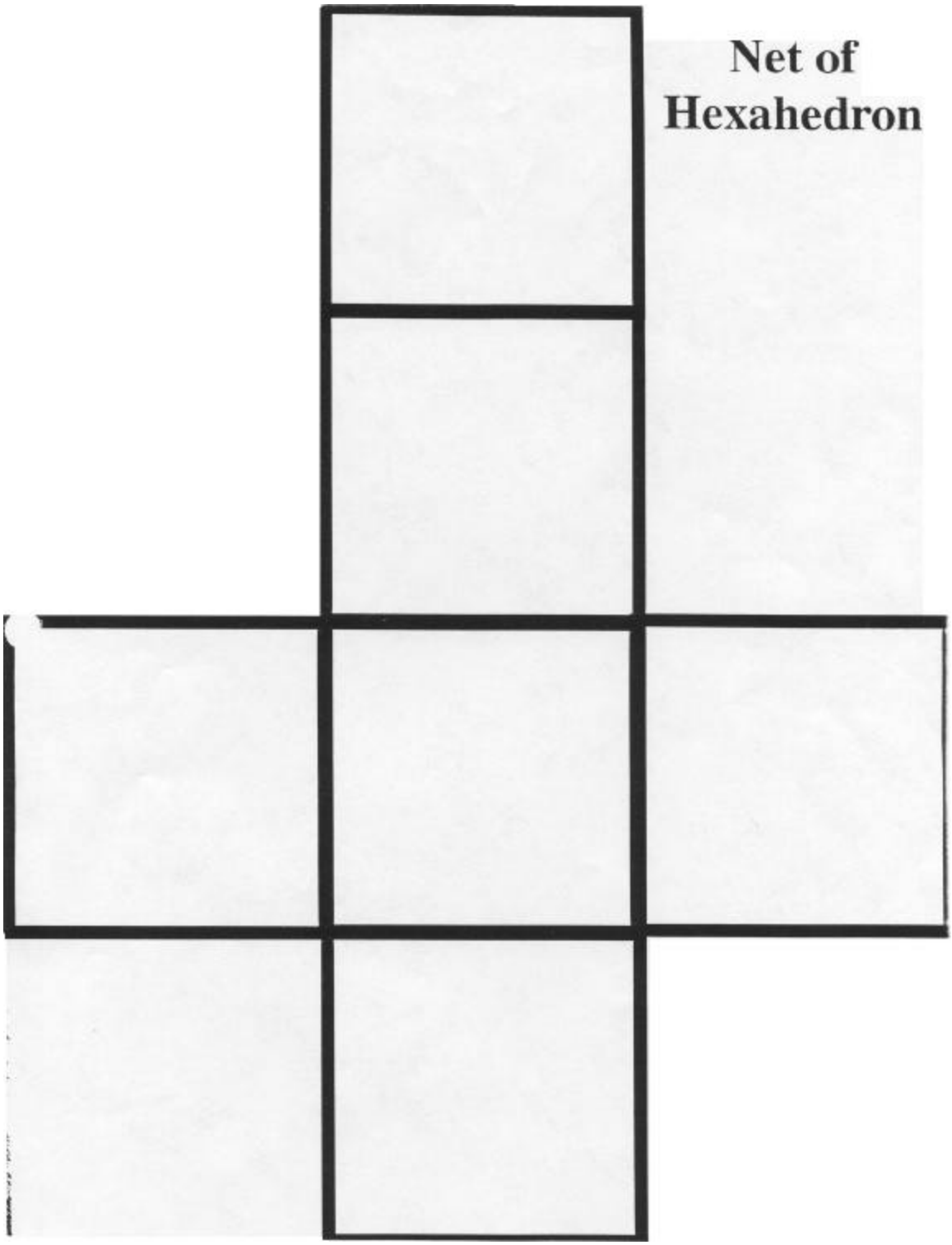
Complete the chart below for five regular polyhedra. *Look for a pattern!*

	Number of Faces	Number of Vertices	Sum of the number of faces & vertices	Number of Edges
Tetrahedron				
Hexahedron				
Octahedron				
Dodecahedron				
Icosahedron				

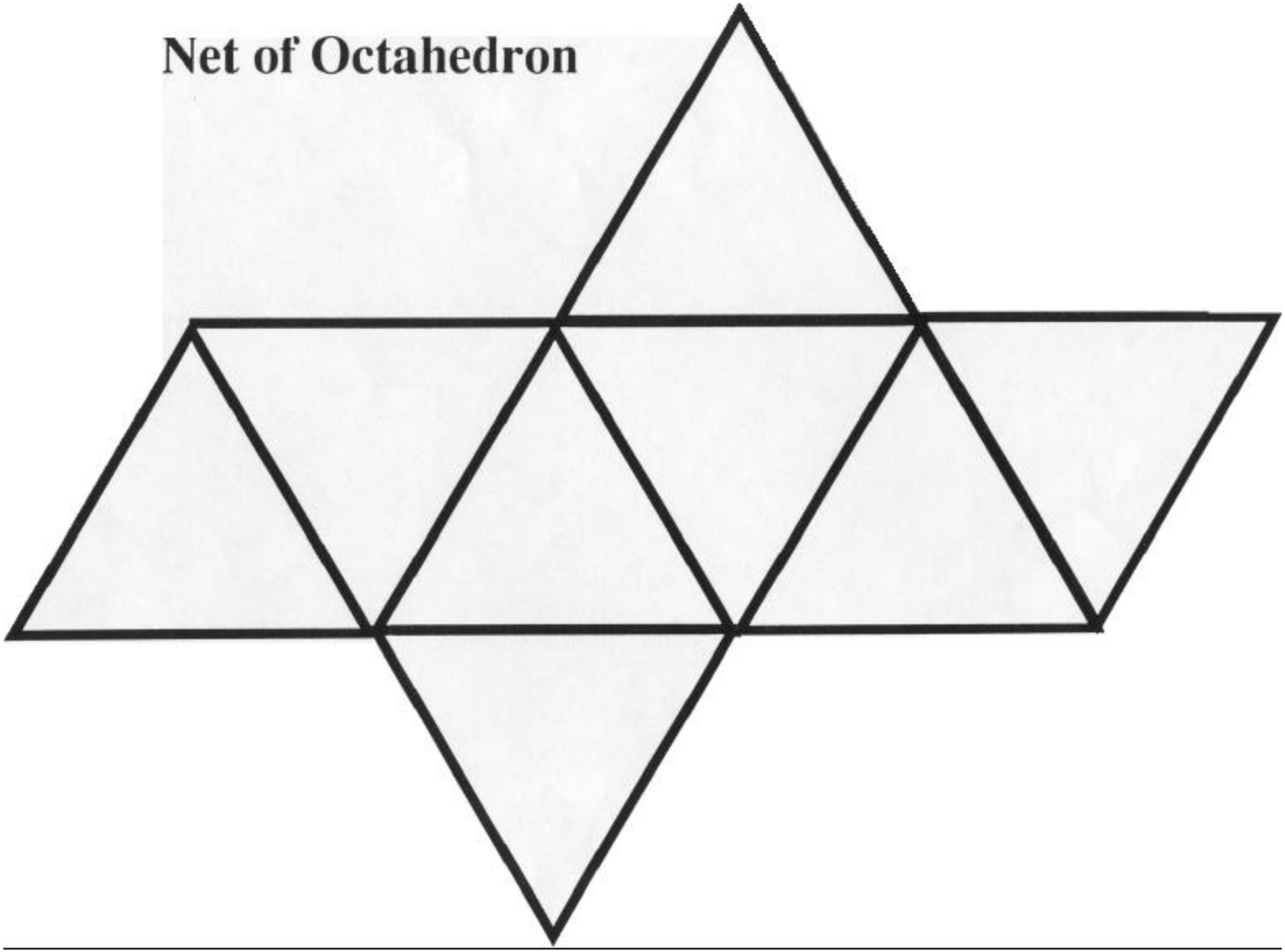
Look carefully at the chart you've just completed. You should be able to find a pattern: the sum of F and V is always _____ more than E or, F + V =_____
This formula is called Euler's formula because the mathematician Leonard Euler developed it.

Net of Tetrahedron

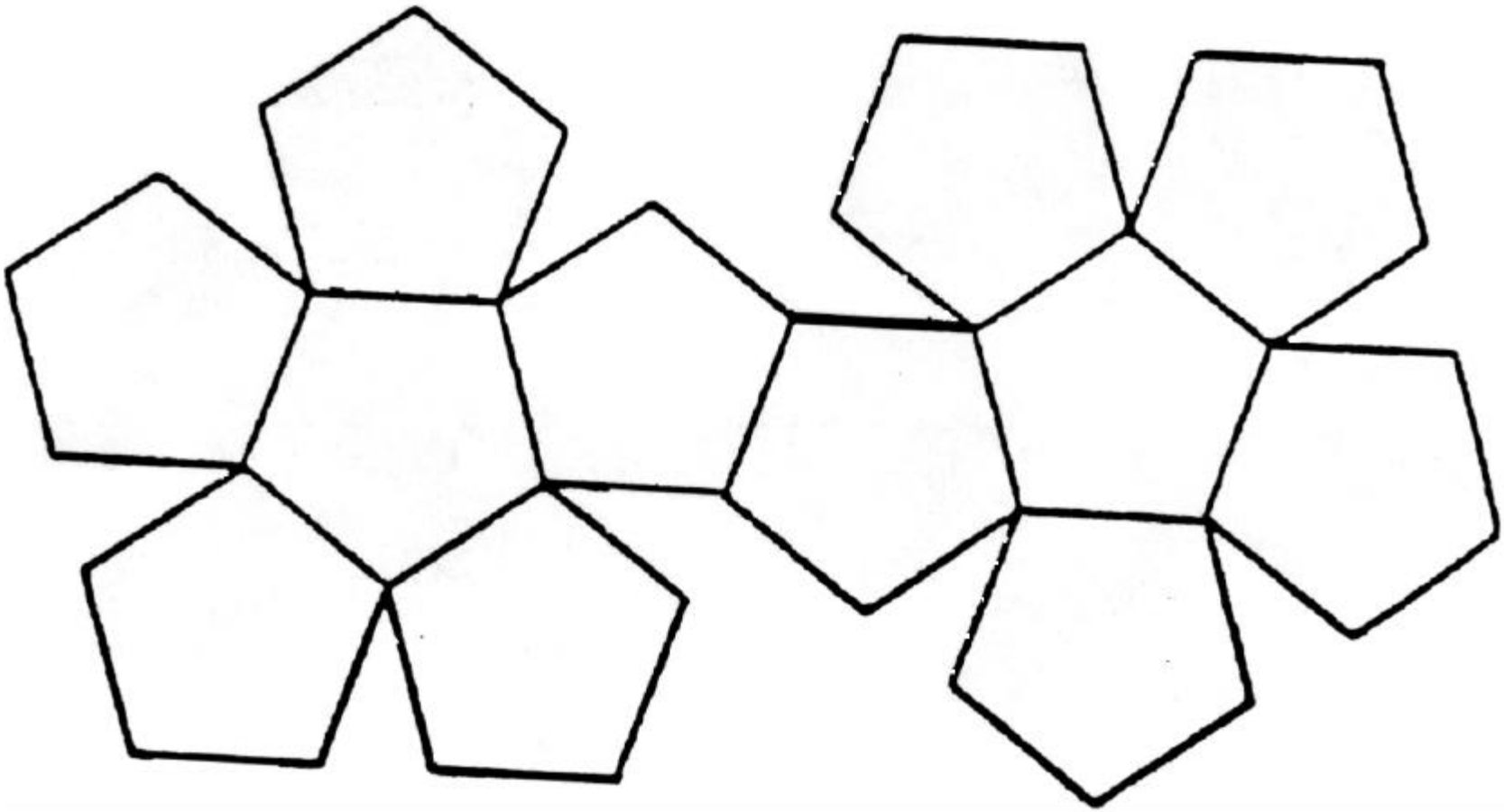



Net of
Hexahedron

Net of Octahedron



Net of Dodecahedron



Net of Icosahedron