PBS Mathline®

The Middle School Math Project

Remove One

(Probability)

Objective

Students will develop winning game strategies based on probable outcomes of events.

Overview of the Lesson

In this lesson, students are given 15 chips and a number line labeled from 2-12. They are instructed to place their 15 chips on the numbers which they think will represent the sums when two dice are rolled. The object of the game is to be the first person to remove all of the chips. A chip is removed if it is on a number that corresponds to the sum rolled. The teacher keeps a record of the sums that are rolled. The game is analyzed and the concepts of probability, sample space and game winning strategies are discussed. Students place chips on their number lines and play the game again. In the video lesson, the number of winners and near winners increased. Finally, the computer is used to show the results which a large number of rolls could yield.

Materials

Per student:

❶ Student Worksheet: Remove One (number line labeled from 2 to 12)

❷ 15 chips per students (beans, buttons, or other counters can be used.)

❸ Handout: Sample Space

Materials *(continued)*

Teacher:

❶ Pair of dice or number cubes (1-6)

❷ Transparency: Remove One

❸ 15 chips

Procedure

Introduce the topic by discussing the possible outcomes that can be obtained when a die is rolled. Students should be able to identify that the possible outcomes are the numbers from 1 to 6. Then ask what are the possible sums if two dice are rolled.

Distribute 15 chips and the Remove One Worksheet, to each student. Instruct them to place their 15 chips anywhere on this number line with the understanding that a chip can only be removed if it is located on a number that represents the resulting sum each time two dice are rolled. Once the students have set up their number lines, roll the dice and call out sums until someone has removed all of his/her chips. Make sure that you keep a record of the sums which are rolled to be used later in analyzing the game. As you are rolling the dice and the students are removing their chips, you may want to check to find out how many removed chips when you call out various numbers (just for the fun of it). Once a winner has been declared, the game is over.

Note: *You may also want to play the game again (before discussion) to determine if students greatly change the placements of their chips by recalling the numbers which were rolled most often during the first game.*

Name the sums which were rolled and have the students place "x's" on their number line to construct a frequency distribution graph. For example, if two fours were rolled the students will place two "x's" by the four on the number line.

Distribute copies of the Sample Space Handout or if you prefer, allow students to create this matrix for themselves. Pose questions which should guide them into discovering that certain sums are more likely to occur than others.

☛ There are 36 different possible outcomes;

☛ From the Sample Space, students should see the pattern: there is one way to roll a sum of 2; there are two ways to roll a sum of 3; there are three ways to roll a sum of 4; there are four ways to roll a sum of 5; there are 5 ways to roll a sum of 6 and there are six ways to roll a sum of 7. At this point the trend reverses: there are five ways to roll an 8; there are four ways to roll a 9; there are three ways to roll 10 , there are two ways to roll 11 and finally there is only one way to roll 12. Depending on the readiness of your students you may wish to introduce "theoretical probability" and "experimental probability" and discuss the following. *See Mathematically Speaking. . .*

After students have had the opportunity to investigate the possible outcomes, instruct them to place their chips on their number lines to reflect their revised winning strategies. You should expect that this analysis may lead them to understand that the probability of rolling a sum of 7 is more likely than the probability of rolling any other specific sum, and that the probability of rolling each sum other than 7 decreases the farther it is from 7. Therefore, you would expect them to place more chips on 7, 6 and 8 as compared to 2, 3, 4, 5, 9, 10, 11, and 12.

Play the game again to determine if there are more winners, if there are fewer rolls and if more students have fewer chips left on their boards when the game is over. If your students do not get better results, extend the discussion to hypothesize why the results were different than expected.

There are software packages or programs which can be incorporated into this lesson to simulate the rolling of two dice, while simultaneously keeping track of the sums. This will allow students to experience what could happen when dice are rolled a large number of times and how close to the theoretical probability the outcomes can be. The video teacher uses one of these programs with his students in this lesson.

Mathematically Speaking. . .

Comparing the number of ways to roll a specific number with the total number in the sample space provides students with the theoretical probability of rolling that specific number. For example, since there are 6 ways to roll a sum of 7. Theoretically, the probability of rolling a sum of 7 is 6 out of 36 or $6/36 = 1/6$; since there is 1 way of rolling a sum of two, the theoretical probability of rolling a sum of 2 is $1/36$. Actual outcomes (experimental probability) may not match the theoretical probability. Students should be encouraged to compare the theoretical probability with the experimental probability.

Extensions & Connections

Create or allow students to create variations of this game, for example, a number line comprised of the numbers from 0 to 9. Again, students will place their chips on the number line, but this time they can only remove a chip if there is a chip on the units digit of the product resulting from the roll of two dice. For example, if the numbers on the faces of two dice are 4 and 5, the product is twenty, and the units digit is 0. If there is a chip on the 0, it can be removed. Have students analyze this game and decide which numbers should theoretically show up most often.

Resources

Probability Lab. MECC, Minneapolis, Minnesota.

Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

Standard 1: Worthwhile Mathematical Tasks

❶ Discuss your feelings about incorporating games involving mathematics as a worthwhile task. Include a brief analysis of a game-playing learning activity which you felt was very successful and/or one that "bombed."

Standard 4: Tools for Enhancing Discourse

❷ Share some information on technology based resources which you have successfully used to assist students in their understanding of probability.

Standard 5: Learning Environment

❸ "Emphasizing reasoning and justification implies that students should be encouraged and expected to question one another's ideas and to explain and support their own ideas in the face of others' challenges." (p. 58) Comment on the value of having students verbalize their reasoning after making their choices.

Standard 6: Analysis of Teaching and Learning

❹ When students make their first selections, what assessment information can the teacher gain about the intuitive understanding of probabilistic concepts? When students revise their selections as they continue playing the game, what additional assessment information can the teacher obtain?

❺ If you use this lesson with your students, what adaptations would you make?

❻ How does this type of learning activity enable the teacher to monitor student understanding, and adjust the teaching of the lesson?

Remove One

12

11

10

9

8

7

6

5

4

3

2

Sample Space

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)

(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)

(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)

(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)

(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)