PBS Mathline®

*The Middle School Math Project*

# Rumors

(Number and Number Relations)

## Objective

Students will explore an exponential growth pattern using problem solving, reasoning, communications and connections.

## Overview of the Lesson

The teacher shares the following scenario with the students:

> Two students who were both born on December 21st, the date of the winter solstice, decide that it would be great not to have to attend school on that day. Therefore, they start a rumor that schools will be closed to celebrate the winter solstice. So, on December 1st, one of the students told two of her friends that school would be closed. On the next day, each of these students tells 2 students and on consecutive days, each of the new students tells 2 more students and so on. If there are 8,000 students in the school district, the question arises as to whether the rumor was started early enough for everyone to have heard it?

Students act out the problem to understand how the rumor is being spread. Next, students work in groups to determine strategies and solutions for solving this and related problems.

## Materials

- Calculators

## Procedure

Begin the class by having the students focus on how rumors are spread. In the video, the teacher whispers something in the ear of one of the students who, in turn, whispers it to her neighbor, and so on.

Next, set up a similar scenario as described below for your class:

> Justin Miller and Agnes Malika share the same birthday on December 21st. One day at the lunch table, they were discussing how great it would be not to have to attend school on that day. Agnes figured that the winter solstice is a day of celebration somewhere in the world, so it should be in New Briton, Connecticut. She decided to start a rumor that all schools in the city will be closed on December 21st. On December 1st, Agnes tells her friends Melissa and Tina. She tells each of them to tell two more students and that each of the new students should tell two more on the following day. There are 8,000 students in the school district.

Allow students to act out this scenario by having students form a human triangle where Agnes is first, then Melissa and Tina, then four student representing the two that Melissa told and the two Tina told, etc. This will help students visualize the problem, understand how this rumor is being spread, and an idea of the growth pattern.

Have students create mathematics questions about this situation. For example: *Was the rumor started early enough for all of the students to have heard that school will be closed on December 21st? On which day would all of the students have heard that school will be closed on the 21st? How many students would have heard the rumor by the 10th day of December?*

Encourage students to apply some of their problem solving strategies (make a diagram, make an organized list, create a rule, etc.) to assist in their construction of solutions to the problems. Tell students that as they work, try to create an algebraic representation or rule which can be used to calculate the number of students who should have heard the rumor on *any* given day.

Place students in groups. Encourage the groups to intuitively arrive at a prediction based on the information obtained from the enactment. Have students devise strategies and make predictions on: *How many students they think should have heard the rumor by the 10th day? On which day could at least half of the students heard the rumor?* Allow an appropriate amount of time for groups to discuss their predictions.

In their groups, allow students to continue their investigation of whether the rumor was started early enough for all of the students in the district to have heard it. They must be able to support their answers. Some students may be able to construct a mathematical rule to symbolize their findings.

Engage students in a class discussion to review their solutions and reasoning. Again, pose questions designed to have students justify their conclusions. These conclusions might be similar to those illustrated in ***Mathematically Speaking . . .***

***Mathematically Speaking . . .***

The chart below provides an organized arrangement of the events which occur in this problem.

| Day | The number of new people who hear the rumor on a given day | Total number of people who have heard the rumor on a given day, including the initiator |
|---|---|---|
| Start of first day | 1 | 1 |
| 1 | 2 | 3 |
| 2 | 4 | 7 |
| 3 | 8 | 15 |
| 4 | 16 | 31 |
| 5 | 32 | 63 |
| 6 | 64 | 127 |
| ? | 256 | ? |
| 10 | 1024 | 2047 |
| 11 | 2048 | 4095 |
| 12 | 4096 | 8191 |
| 13 | 8192 | 16383 |
| 14 | 16384 | 32767 |
| • | • | • |
| ? | 65536 | ? |
| • | • | • |
| ? | ? | 524287 |
| • | • | • |
| 20 | ? | ? |
| • | • | • |
| $n$ | $2^n$ | $2^{n+1} - 1$ |

Guide students through the chart, assisting them with understanding each entry. Have them first look at the chart from merely a computation viewpoint and then, if appropriate, from an algebraic viewpoint. When you observe what is happening in each of the columns, you will see that in the first column, if the day is increased by 1, the rumors double in the second column, and the sum is almost doubled in the third column. (This is the type of observation that encourages students to focus on the relationship between two quantities and develop intuition about functional relationships. The relationship between the number of days and the number of rumors is an exponential function.

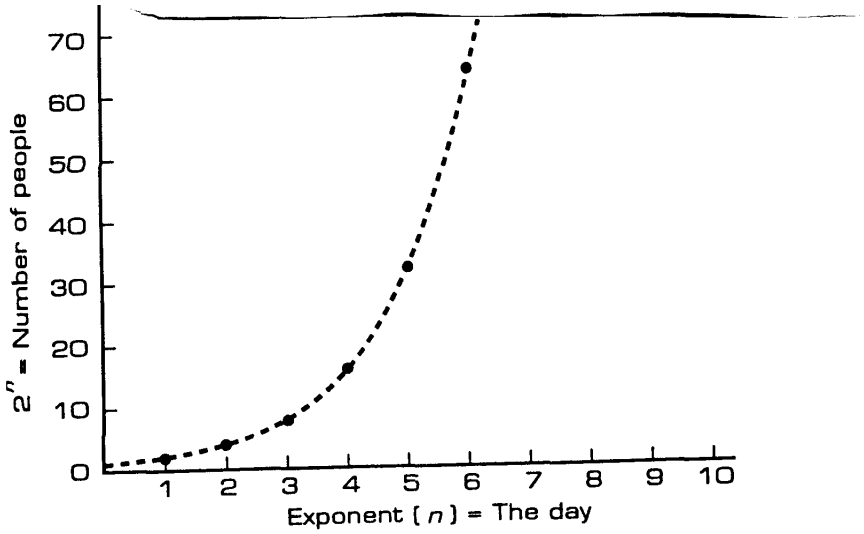
From the chart, it is important to note that before the first day, only one person thought of initiating the rumor (excluding the other student who was involved). If $n$ represents the day, then by the end of Day 1, $2^n$ represents the number of new people who have heard the rumor on day $n$ and $2^{n+1}$-1 represents the total number of people who knew the rumor on a given day (including the person who started the rumor).

By examining the pattern in the second column — the powers of 2 — the question is therefore, how many twos are in each entry? [2 =2; 4=2 x 2 ; 8 = 4 x 2 = 2 x 2 x2 ; etc.] Another way to write this is $2 = 2^1$ ; $4 = 2^2$; $8 = 2^3$; $1024 = 2^{10}$ (The 2 is called the base and the

10 is called the exponent. The exponent indicates how many times the base is used as a factor.

## Extensions & Connections

Have students create and analyze the graph of the growth pattern in this lesson. For example, below is the graph of the "number of new people who hear the rumor on a given day" or $2^n$.




**Number of people who hear the rumor on a given day**

You may also have the students create the graph of the total number of people who knew the rumor by the end of any given day" or $2^{n+1}$-1. They can compare/contrast the two graphs.

Have students compare/contrast the graph of a linear growth pattern with the graph of an exponential growth pattern.

You might want to introduce this problem: A riverboat gambler plays "double or nothing." He is down to his last dollar. On what hand would he be a millionaire if he won every hand? You might want to set up a table with the headings "Hand Number" and "Dollars won after hand was played." Remember in hand #1, the dollar entry is $1.

## Resources

National Council of Teachers of Mathematics. *Curriculum and Evaluation Standards for School Mathematics Addenda Series: Patterns and Functions.* (1993) Reston, Virginia.

## Ideas for Online Discussion

(*Some ideas may apply to more than one standard of the* ***NCTM Professional Standards for Teaching Mathematics.)***

### Standard 1: Worthwhile Mathematical Tasks

❶ The *Standards* state that a task can engage students intellectually if it challenges them to search for something. Reflecting on the *Rumors* lesson, did you feel the students were challenged to "search for something." Outline a lesson that you feel challenges students to "search."

### Standard 4: Tools for Enhancing Discourse

❷ One way to enhance discourse is to have students act out the problem. To an extent this was done in *Rumors*. How have you used dramatizations in math?

### Standard 5: Learning Environment

❸ When you pose questions to the class and one or two students respond, what techniques do you use to ensure/assess that the other students also understand the concepts and are focused on the lesson.

### Standard 6: Analysis of Teaching and Learning

❹ If you selected one of your students to replace you as the teacher for a day, more than likely he or she will be a "young" clone of you. What attributes of your teaching style would you most like to see emerge from this student as he assumes the role of teacher?

❺ The growth pattern in this lesson might be pretty difficult for students to readily understand. Share a couple of methods you use to reteach/review concepts when students "just don't understand."

❻ There may be many ways to solve any given problem and students are masters at finding some of the most unique and extensive solutions to some of the problems that are assigned. Many times their processes are very time consuming and disorganized. Do you encourage your students to use more traditional problem solving processes or are any methods that they use acceptable?