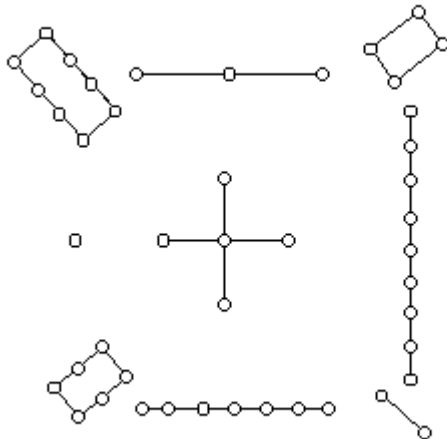


## Activity 2: Magic Squares and Stars

### Magic Squares

For centuries, mathematicians and individuals interested in recreational mathematics have been interested in magic squares. Practically all historians agree that the magic square had its origin in China centuries before the birth of Christ. The exact origin has been lost, but Oriental tradition holds that the Emperor Yu (c.2200 B.C.) was standing on the bank of the Yellow river when a tortoise appeared with a mystic symbol on its back. This figure came to be known as the *lo-shu*, and is shown below.



The *lo-shu* consists of a 3 x 3 square of numbers, indicated by knots tied in a string, and so arranged that the sum of the number of knots in any row, column, or diagonal is fifteen. In decimal representation, it appears as the following figure:

4	9	2
3	5	7
8	1	6

The square is “magic” because the sum of any row, column, or diagonal is the same.

The magic square is still common in China today. It is found on buildings and in artistic designs, and fortune tellers use them in their trade.

After many centuries, the magic square found its way out of China. In the ninth century, magic squares were used by Arabian astrologers to read horoscopes. The magic square appeared in India in the eleventh and twelfth centuries. The figure below shows a magic square found in the ancient town of Khajuraho.

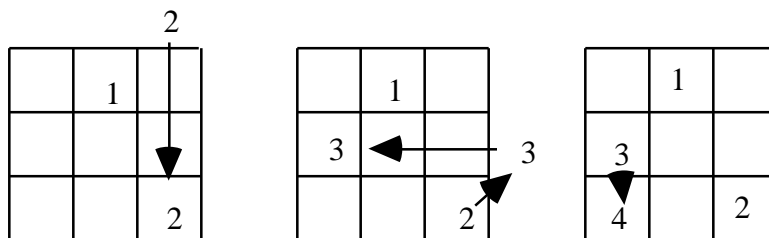
7	12	1	14
2	13	8	11
16	3	10	5
9	6	15	4

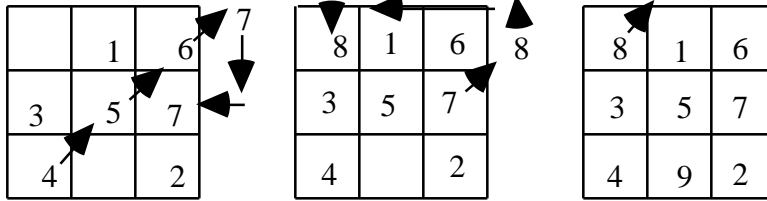
### Constructing Magic Squares

The method for constructing magic squares of any order is called the La Loubre method. As a demonstration, here are the steps for constructing a magic square of order three.

1. Place the successive numbers in the cells in their natural order in a diagonal line that slopes upward to the right. Begin with 1 in the center of the top row.
2. Any time you reach the top row, write the next number in the lowest cell of the bottom row of the next column on the right.
3. When you reach the right-hand column, write the next number in the left-hand column as if it immediately succeeded the right-hand column.
4. When you reach a cell that is already filled, or when you reach the right-hand upper cell, write the next number in the cell directly below the last number written.

### Example of Rules





You can only use the La Loubre method to construct odd order magic squares (3x3, 5x5...). For this activity, your work is limited to odd order magic squares. Check the resources for books to further study magic squares of either order.

1. Below is a magic square of order 7. The sum of the rows, columns, and diagonals is 175. Find the missing number from the rows or columns.

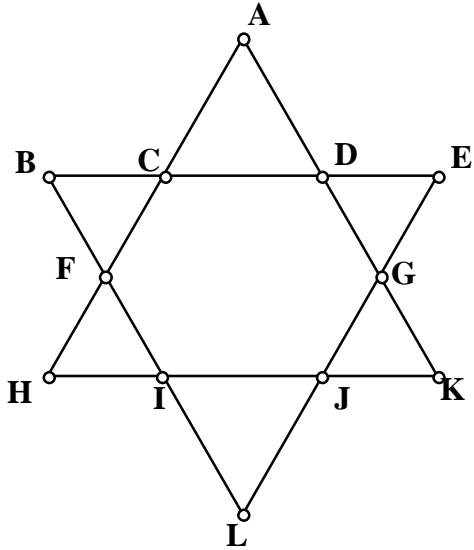
30	39		1	10	19	28
38	47	7	9		27	29
	6	8	17	26	35	37
5	14	16	25	34		45
13		24	33	42	44	4
21	23	32	41	43	3	
22	31	40		2	11	20

2. Construct a magic square of order five.

### Magic Stars

Today we have other puzzles that are related to magic squares. Mutsumi Suzuki is a Professor of Engineering in the Laboratory for Process Systems Engineering, Tohoku University, Sendai, Japan. His research interests include reduced gravity and chemical engineering and process system engineering. Magic Stars are one of his hobbies.

A magic star is a six-sided star with numbers at each of its vertices. The sum of the numbers along each segment is a constant. Consider the following diagram.



Edge  $A + C + F + H = \text{edge } A + D + G + K = \text{edge } B + C + D + E = \text{edge } B + F + I + L$   
 $= \text{edge } E + G + J + L = \text{edge } H + I + J + K = N$  (constant)

Each letter represents a number from 1 to 12.

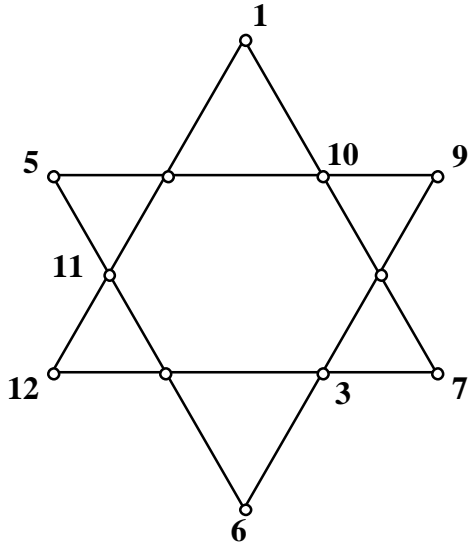
therefore  $1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 = 78$   
 $A + B + C + D + E + F + G + H + I + J + K + L = 78$

now

$$\begin{aligned}
 &\text{edge } A + C + F + H = N \\
 &\text{edge } A + D + G + K = N \\
 &\text{edge } B + C + D + E = N \\
 &\text{edge } B + F + I + L = N \\
 &\text{edge } E + G + J + L = N \\
 + &\text{edge } H + I + J + K = N \\
 \hline
 \end{aligned}$$

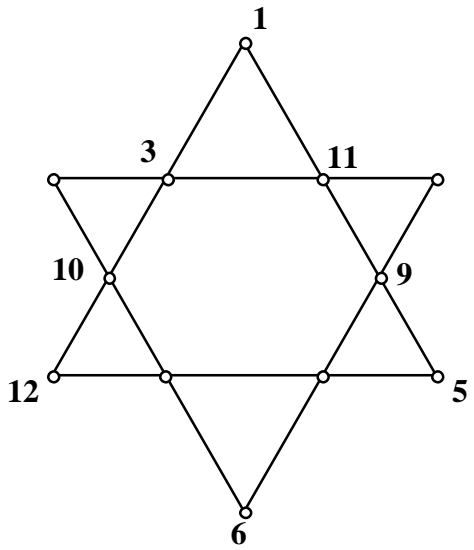
$$\begin{aligned}
 2A + 2B + 2C + 2D + 2E + 2F + 2G + 2H + 2I + 2J + 2K + 2L &= 6N \\
 2(A + B + C + D + E + F + G + H + I + J + K + L) &= 6N \\
 2(78) &= 6N \\
 156 &= 6N \\
 156/6 &= N \\
 26 &= N \\
 156 &= 6N \\
 156/6 &= N \\
 26 &= N
 \end{aligned}$$

3. Below is a magic star with numbers missing. Fill in the missing numbers.

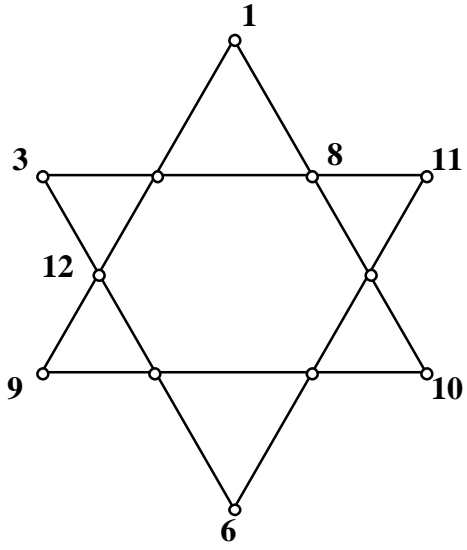


4. Complete the following the magic stars below by supplying the missing numbers.

a.



b.



c.

