April 2000

Activity 2: Pythagoras and President Garfield

Solutions

A. The Pythagorean Theorem

1. a. $\sqrt{8^2 + 9^2} \approx 12$

b. $\sqrt{13^2 - 5^2} = 12$

2. Since $12.5^2 + 15^2 = 381.25$ and $20^2 = 40{,}0$ the corners are not square. The angle between the two shorter sides needs to be reduced until the long side is approximately 19.5.

B. The President's Proof

3. They are congruent. By construction, side BC and CE are the same. Angle ECD is the same as angle ABC because exterior angle ECD + the Right Angle = Angle ABC + Right Angle interior to triangle ABC. Since the hypotenuse and an angle are congruent, the two triangles are congruent. Now they have CD = x and DE = y

4. a. ½ x y

b. ½ z z = ½ z^2

c. ½ xy

5. Trapezoid

6. Area of a trapezoid is 1/2 the height times the average of the bases so,
½ (x+y)(x+y) = ½ (x^2 + 2xy + y^2) = ½ x^2 + xy + ½ y^2

7. They are equal since they occupy the same space.

8. $\frac{1}{2} x y + \frac{1}{2} z^2 + \frac{1}{2} x y = \frac{1}{2} x^2 + xy + \frac{1}{2} y^2$

9. $xy + \frac{1}{2} z^2 + = \frac{1}{2} x^2 + xy + \frac{1}{2} y^2$
$\frac{1}{2} z^2 = \frac{1}{2} x^2 + \frac{1}{2} y^2$
$z^2 = x^2 + y^2$

10. Students have proven the Pythagorean theorem.