April 2000

Activity 2: Pythagoras and President Garfield

A. The Pythagorean Theorem

The Pythagorean theorem is one of the most well known and important theorems in all of mathematics. It is very useful for finding a side of a right triangle when only two are known. It is also useful to help one establish whether or not a triangle is a right triangle (if it has a 90-degree angle).

The Pythagorean theorem states that a triangle is a right triangle if and only if the square of the longest side equals the sum of the squares of the two other sides. In other words, the triangle in Figure 1 is a right triangle if and only if $x^2 + y^2 = z^2$.

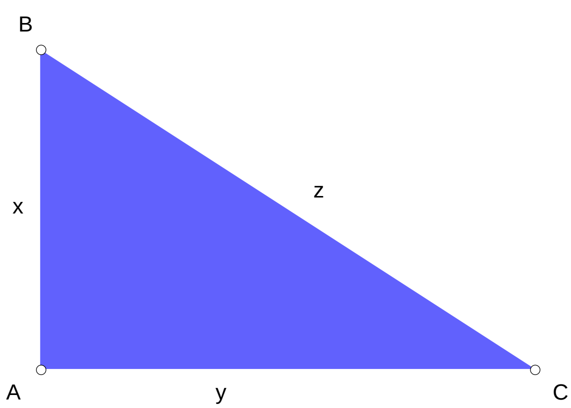
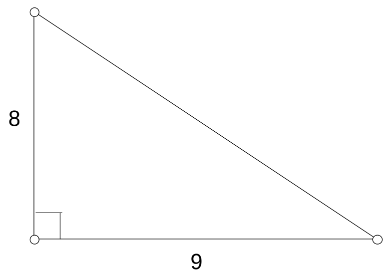


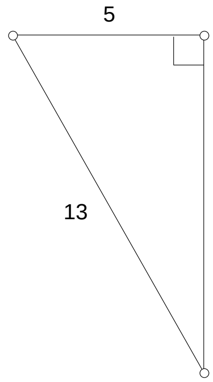
Figure 1. A triangle.

1. Use the Pythagorean theorem to find the unknown side of each triangle.

a.



b.



2. A builder is framing a house. She wants to make sure the walls are at right angles to each other. She makes the measurements of two walls and the diagonal, as shown in Figure 2. Use the Pythagorean theorem to help the builder decide if the walls are at right angles.

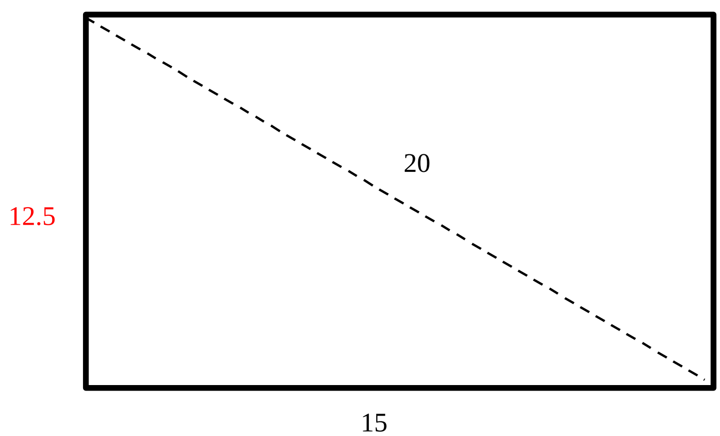


Figure 2. Builder's diagram.

B. The President's Proof

There are many proofs to the Pythagorean theorem. President James Garfield developed his own proof in *The Journal of Education* (Volume 3 issue161) in 1876. President Garfield studied math at Williams College (in Williamstown, MA) and taught in the public school in Pownal, Vermont, for a year or two after graduating. President Garfield may have been joking when he stated about his proof that, "we think it something on which the members of both houses can unite without distinction of the party." A nice feature of mathematical proofs is that they are not subject to political opinion.

See if you can follow the president's proof.

See Figure 3. Starting with the right triangle ABC, construct a line perpendicular to BC and through the point C. Extend AC from the point C.

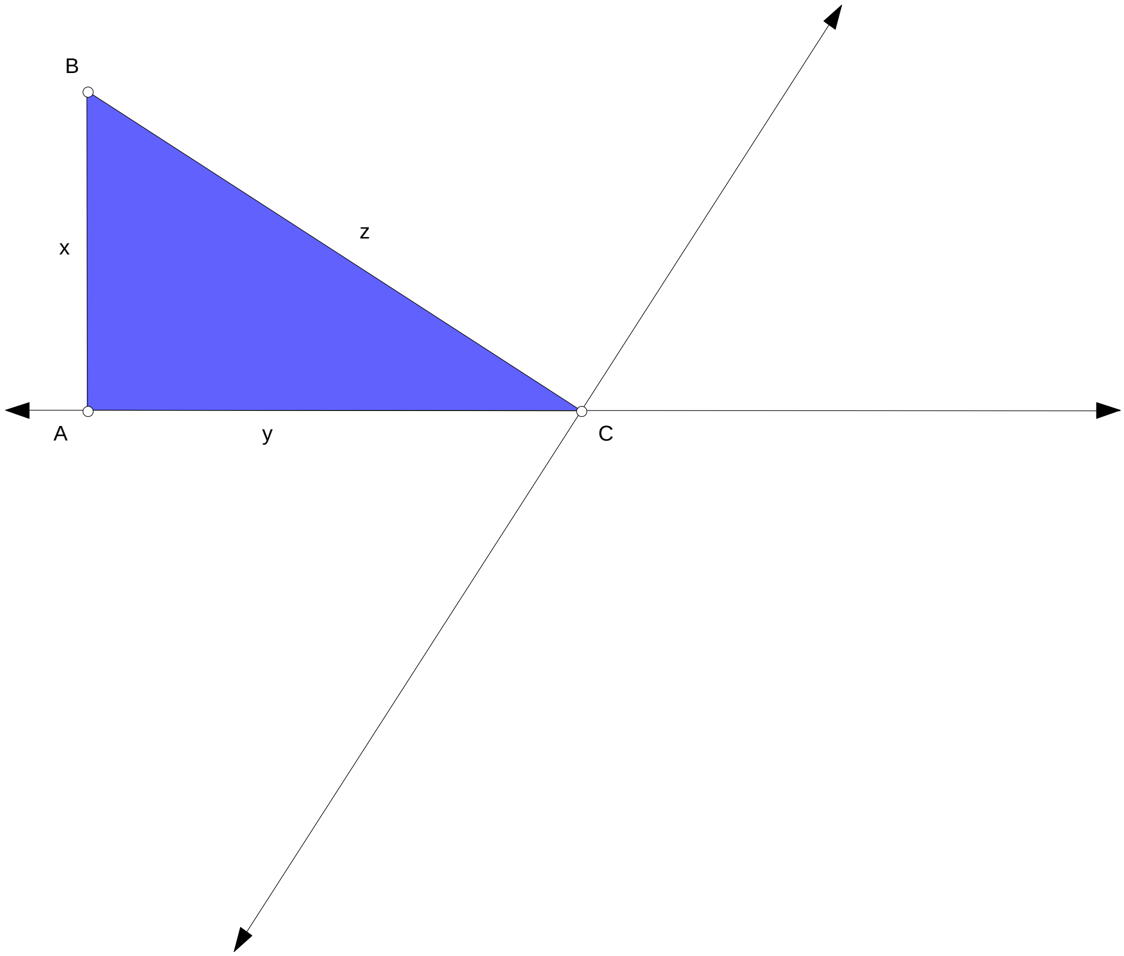


Figure 3. Diagram of triangle ABC for use in the president's proof.

Now look at Figure 4. Draw a circle with center C and radius equal to the length of BC. Label the point E as the point where the circle and the perpendicular line intersect. Draw a line through E and perpendicular to AC and label the intersection point D. Lastly, draw the segment BE.

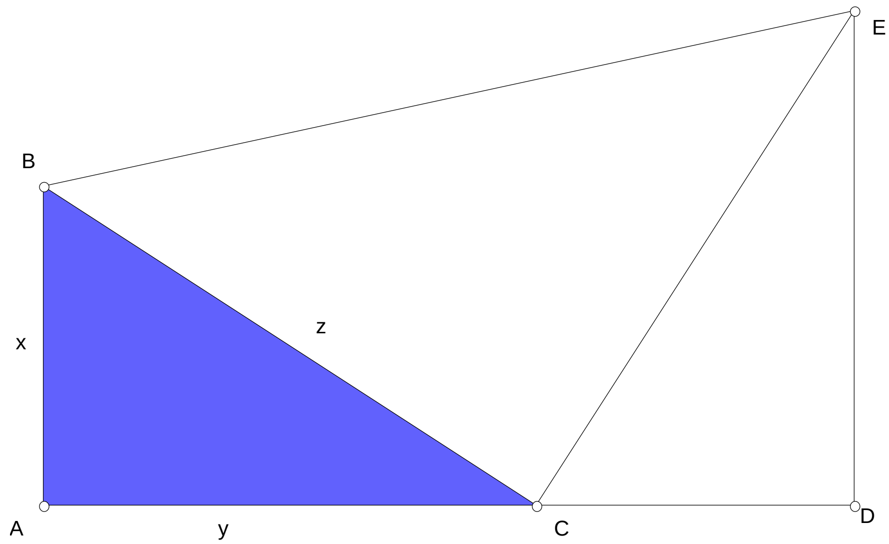


Figure 4. Diagram for use with the president's proof.

3. What can you say about triangles ABC and CDE?

4. What is the area of

a. triangle ABC?

b. triangle BCE?

c. triangle CDE?

5. What type of quadrilateral is ABDE?

6. What is the area of ABDE?

7. What can be said about the sum of the area of triangles ABC, BCE, and CDE and the area of ABDE?

8. Using only *x*'s, *y*'s and *z*'s, write an equation for the sum of the area of triangles ABC, BCE, and CDE and for the area of ABDE.

9. Solve the equation you found in problem 8.

10. What have you proven?