

Squaring Off With Velocity

Activity Summary

Students investigate the meaning of c^2 in $E=mc^2$ by measuring the energy delivered by an object falling at different velocities. Graphing data leads students to understand that E is proportional to velocity squared, not simply velocity.

Materials for each team

- copy of “Squaring Off With Velocity” student handout
- copy of “Data Sheet” student handout
- 1 lb flour in a plastic bag
- plastic pan (about shoebox size)
- two 8-oz plastic cups
- four standard-sized (about 1 cm) glass marbles
- plastic ruler
- meter stick
- plastic spoon
- wood dowel or wood barbecue skewer
- felt-tip pen
- graph paper
- newspaper

Background

Light has fascinated scientists for centuries. Galileo Galilei was the first to consider measuring its speed. In 1676, astronomer Ole Roemer made observations of the eclipses of Jupiter’s moons to demonstrate that light moved at a very fast—but not infinite—speed. James Clerk Maxwell provided the mathematical backbone for electromagnetism and demonstrated that light was an electromagnetic wave. The squared part of Albert Einstein’s equation heralds back to natural philosopher Gottfried Leibniz, who proposed that an object’s energy is the product of its mass times its velocity squared, not just its velocity. Emilie du Châtelet further championed his ideas. While many of these scientists were innovative thinkers, determination also played a large part in their achievements. They were willing to challenge widely held beliefs of their day. Their courage and perseverance helped lay the groundwork for Einstein’s eventual connection of mass, energy, and the speed of light squared.

In this activity, students use a simple model to investigate the relationship between velocity and energy. Their investigation leads them to conclude that the energy delivered to a system depends on the velocity squared of the impacting body, not simply on the velocity. Students then relate this fact to $E=mc^2$.

LEARNING OBJECTIVES

Students will be able to:

- explain what the c^2 in $E=mc^2$ represents.
- state that kinetic energy is the energy of an object in motion.
- illustrate how kinetic energy can be transferred to other objects.
- understand that the energy delivered by an object in motion is proportional to v^2 , not v .

KEY TERMS

kinetic energy: Energy of a moving object.

speed: The rate at which an object moves.

velocity: The speed and direction of a moving object.



STANDARDS CONNECTION

The “Squaring Off With Velocity” activity aligns with the following National Science Education Standards (see books.nap.edu/html/nses) and Principles and Standards for School Mathematics (see standards.nctm.org/document/index.htm).

GRADES 5–8

Science Standard

Physical Science

- Transfer of energy

Mathematics Standard

Measurement

GRADES 9–12

Science Standard

Physical Science

- Conservation of energy and the increase in disorder

Mathematics Standard

Measurement

The model uses a glass marble as a falling object that impacts a cup of flour. The impact velocity of the marble is a function of the height from which the marble is dropped. The energy released by the falling marble (its potential energy now turned into kinetic energy) is equal to the work done on the flour. The work done on the flour, in turn, is equal to the force (mass x deceleration) it takes to slow the marble down to zero velocity over the distance it penetrates the flour.

Four heights are used—10, 25, 50, and 100 centimeters. Students graph the velocity the marble attains when dropped from these heights against the depth to which the marble penetrates the flour. The depth, in turn, is a measure of the energy that the marble delivers. Students can calculate the impact velocity at these four heights by using the equation:

$$\nu = \sqrt{2gd}, \text{ where } g = \text{the acceleration due to gravity (9.8 m/sec}^2\text{),}$$

and d = the distance the marble falls (in meters)

Procedure

- 1 Assemble all the materials needed for the activity.
- 2 Write $E=mc^2$ on the board and ask students what the three letters in the equation represent. Emphasize that c stands for a particular constant speed or velocity, that of light in a vacuum.
- 3 Demonstrate the parts of the apparatus students will use to find the relationship between E and ν (a replacement in the model for c). Marbles must be dropped from rest, and the depth to which the marbles penetrate must be measured with a dowel or skewer marked off in centimeters. The depth should be measured from the top of the marble to the top of the flour. The cup must be picked up, and the skewer must be viewed from the side to measure depth accurately. Student should add a half centimeter to their measurements in order to measure to the marble’s center of mass.
- 4 Organize students into teams. Distribute the student handouts and materials. Assign teams to one of the four heights from which to drop marbles. To ensure data reliability, have several teams perform the same measurements multiple times and average the results. As a class data table is going to be made, the more data points the better.
- 5 Have students place newspaper on the floor, then place their cups filled with flour in the center of a plastic pan. If students run out of clear area in which to drop the marble, have them scoop out the marble(s) with the plastic spoon, use the spoon to refill the cup, tap the base of the cup three times to remove air pockets, and then use the dowel to level the flour to the cup’s rim. If students’ fingers plunge in to retrieve marbles, they will pack down the flour and their next set of data will be skewed with lower penetrating distances. Results will also be skewed if they leave air pockets in the flour.
- 6 Ask students how they might find the velocity of the marble as it hits the flour. When they arrive at the correct mathematical strategy ($\nu = \sqrt{2gd}$), have students calculate the velocity values for the four given heights. (Students will need to convert centimeter drop heights to meters.) Students may notice that doubling the height from which

they drop the marble does not double the velocity. If the calculations are too rigorous for students, provide them with the values for velocity (see Activity Answer on page 23) and tell students to square them for the v^2 column of their table.

- 7 When student teams are finished, create a class data chart on the board and have students fill it in or have them enter their data into a computer spreadsheet.
- 8 This is an ideal time to do some data analysis and statistics. Student answers may vary quite a bit. For a given distance, ask students which of the data points are “wrong” and which ones are “right.” Discuss the best way to average the numbers so students can graph just one depth for each distance dropped. Cross out the two highest and lowest points (outliers) and average the rest. Have students average all teams’ depth results to determine final class averages for each depth. Have students enter the results in the depth column in the “Velocity vs. Energy Data Table” on their “Data Sheet” handout.
- 9 Discuss factors that may cause data to vary, i.e., non-uniform density of the flour, problems with measuring depth, variations in tick marks, etc. Emphasize that while accuracy is important, measurements may include a degree of error. The goal is to see the pattern in the data set (that energy is proportional to velocity squared, not velocity).
- 10 Tell students they will make two graphs, the first with the y -axis labeled Velocity (meters/second) and the x -axis labeled Energy (depth in centimeters). The second graph will have the y -axis labeled Velocity² (meters²/second²) and the x -axis labeled Energy (depth in centimeters). Meters are used on the velocity axes to simplify graphing. Note that 0,0 *must* be used as a data point when drawing the curve.
- 11 Have students plot the first graph. When students have finished plotting points, review how to interpolate and draw a curve through a set of points instead of drawing “dot to dot,” as students will often do. Discuss the fact that a curve shows that the two variables are not directly proportional.
- 12 Now have students plot their second graph in which the velocity is squared. After students finish their second graph, help them draw a straight line through as many points as possible. They should try to have roughly the same number of points on either side of their lines. Then have student teams answer the questions on their “Data Sheet” handout. Review answers as a class. What is the most noticeable difference between the two graphs?
- 13 As an extension, have students use $v = \sqrt{2gd}$ to calculate from what heights marbles would need to be dropped to double the velocity for each height (beginning with the 10-centimeter height). Have the students repeat the experiment at this new set of heights (you may need to use very wide-mouth cups in order for students to hit the target flour). Extend the heights above the flour to 2 meters. Stop taking data when the marble hits the bottom of the cup. Direct students to plot this new set of data, compare it to their previous graphs, and find the slopes for each line. Are they the same? Have students explain their results.

THE NEED FOR C

c is necessary in Einstein’s equation because whenever part of any piece of matter is converted to pure energy, the resulting energy is by definition moving at the speed of light. Pure energy is electromagnetic radiation—whether light or X-rays or whatever—and electromagnetic radiation travels at a constant speed of about 300,000 kilometers per second.

The speed of light must be squared because of the nature of energy. When something is moving four times as fast as something else, it doesn’t have four times the energy but rather 16 times the energy—in other words, that figure is squared. So the speed of light squared is the conversion factor that determines just how much energy lies within any type of matter.

ACTIVITY ANSWER

Marbles dropped from different heights accelerate toward the surface of the flour, increasing their velocity and kinetic energy as they fall. The kinetic energy that the marble has gained is then transferred to the flour as it plunges in. The depth that the marble reaches in the flour is a measure of the kinetic energy that is transferred to the flour (the energy deforms the flour and makes the marble crater).

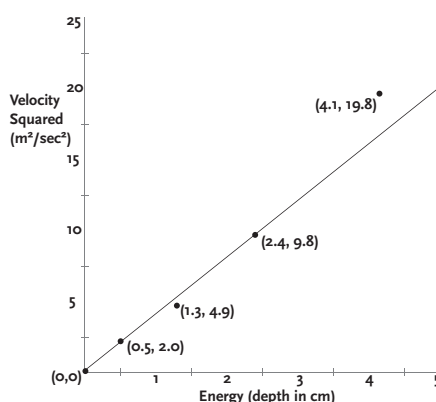
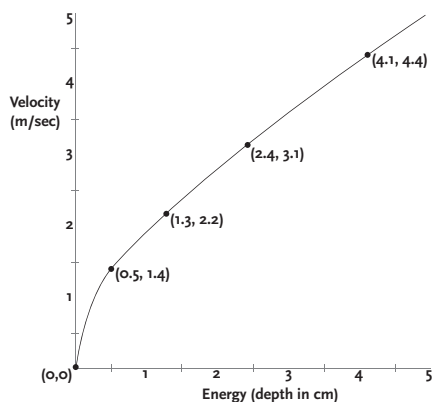
Student results may vary due to differences in flour density and errors in dropping the marble from prescribed heights. When reviewing the table with students, it would be best to eliminate the two lowest and highest values for each height and average the rest. The summary data plotted should reveal that energy is proportional to velocity squared.

Sample Results

Velocity vs. Energy Data Table

Distance (cm)	v (m/sec)	v^2 (m ² /sec ²)	Depth (cm)
0	0	0	0
10	1.4	2.0	.5
25	2.2	4.9	1.3
50	3.1	9.8	2.4
100	4.4	19.6	4.1

The average error of the depth data was about +/- .5 cm



Student Handout Questions

- 1 What is the shape of your Velocity vs. Energy graph?
The shape of the graph is a curve.
- 2 What is the shape of your Velocity² vs. Energy graph?
The shape of the graph should be close to a straight line.
- 3 If a straight line on a graph indicates a direct relationship, is energy (measured by depth) directly proportional to velocity or velocity squared? The Velocity² graph appears to show that energy is proportional to velocity squared, not velocity.
- 4 Explain why Albert Einstein wrote $E=mc^2$ instead of $E=mc$.
Students may forget that the c is simply the velocity of light. Einstein wrote c^2 because E is directly proportional to velocity squared, not velocity.

LINKS AND BOOKS

Links

NOVA—Einstein's Big Idea

www.pbs.org/nova/einstein

Hear top physicists explain $E=mc^2$, discover the legacy of the equation, see how much energy matter contains, learn how today's physicists are working with the equation, read quotes from Einstein, and more on this companion Web site.

Answers from Scientists

www.skirball.org/exhibit/einstein_answers_light.asp

Answers several questions related to light and $E=mc^2$.

The Electromagnetic Spectrum

imagers.gsfc.nasa.gov/ems/waves3.html

Describes the electromagnetic spectrum and includes information on visible light.

The Ultimate Physics Resource Site

serendip.brynmawr.edu/local/IIT/projects/Glasser.html

Includes physics links and activities.

Books

Energy

by Jack Challoner.

Dorling Kindersley, 1993.

Surveys various sources of energy and the ways in which they have been harnessed.

Light

by David Burnie.

Dorling Kindersley, 1999.

Explains many aspects of visible light and other forms of electromagnetic energy.

Stop Faking It!: Light

by William C. Robertson.

NSTA Press, 2003.

Provides information and activities to help teachers and students understand light.

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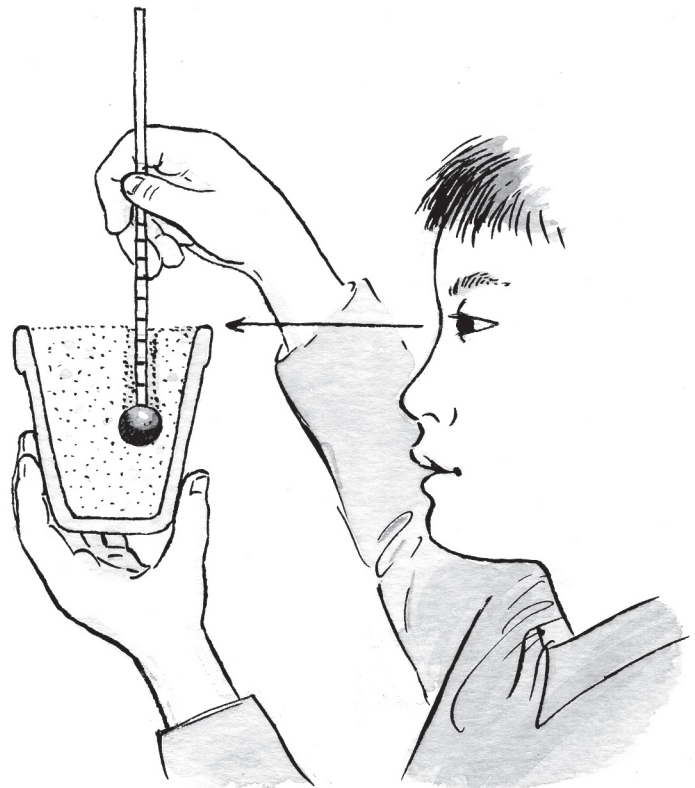
c is the velocity of light in a vacuum. Why is c^2 part of Albert Einstein's equation and not just c ? In this activity, you will measure the energy delivered to an object (flour) by another object (a marble) to determine the relationship between energy and velocity as expressed in $E=mc^2$.

Procedure

- 1 Working over your pan, use the spoon to fill both of your cups a little above the rim with flour. *Do not pack the flour down.*
- 2 Tap the base of the cup three times to remove air pockets. Use your wood dowel or skewer to even off the flour with the top edge of the cup.
- 3 Use a felt-tip pen and plastic ruler to mark ten 1-centimeter tick marks on the wood dowel or skewer. You will use this to measure the depth of penetration by your marble.
- 4 Your class will be dropping the glass marbles into the flour from 10, 25, 50, and 100 centimeters. Your teacher will tell you which part of the class data your team will be responsible for gathering.
- 5 Use your meter stick to measure your assigned height above the rim of the cup. Drop a marble into the cup from that height (see illustration on your "Data Sheet" handout). Make sure the marble does not strike the side of the cup.
- 6 Pick up the cup and place it at eye level as you use the wood dowel or skewer to measure the depth of penetration from the top of the marble to the top of the flour. Record the depth of the hole made by the marble to a tenth of a centimeter. Add half the marble's diameter (.5 centimeter for a standard-sized marble) to your measurement in order to measure to the marble's center of the mass.
- 7 Repeat this procedure at the same height three more times. If you run out of clear space in which to drop your marble, use your spoon to retrieve your marble(s) and refill the cup with flour. Tap the cup three times after it has been refilled and use the wood dowel or skewer to level off the top.
- 8 Add your data to the class data table.
- 9 After your class determines the average depth at each height, enter the depths in the table on your "Data Sheet" handout. Follow the instructions on that handout to create your graphs.

Measuring Energy

Energy is usually measured in joules. In this experiment, you will be measuring energy by the depth of the crater the marble creates in the flour. Then you will be graphing depth (energy) against the velocity at which the marble falls over a certain distance.



Data Sheet

Procedure

- 1 Calculate the velocity of the marble as it hits the surface of the flour. Enter your calculations in the data table below.
- 2 Make a graph with Velocity (meters/second) on the y-axis and Energy (depth in centimeters) on the x-axis. Plot the velocity and depth data you have entered into the table below.
- 3 Draw a smooth curve or line through as many points as possible. Be sure to start drawing your line at the origin 0,0.
- 4 Square your velocity values and enter them in your data table. Draw a new graph with Velocity² (meters²/second²) on the y-axis and Energy (depth in centimeters) on the x-axis. Plot the velocity² and depth data from the table below.
- 5 Draw a smooth line through as many of the points as possible, passing your line above or below points that do not line up exactly. Start your line at the origin 0,0.
- 6 Answer the questions below.

Questions

Write your answers on a separate sheet of paper.

- 1 What is the shape of your Velocity vs. Energy graph?
- 2 What is the shape of your Velocity² vs. Energy graph?
- 3 If a straight line on a graph indicates a direct relationship, is energy (measured by depth) directly proportional to velocity or velocity squared?
- 4 Explain why Albert Einstein wrote $E = mc^2$ instead of $E = mc$.

Velocity vs. Energy Data Table

Distance (cm)	v (m/sec)	v^2 (m ² /sec ²)	Depth (cm)
0			
10			
25			
50			
100			

