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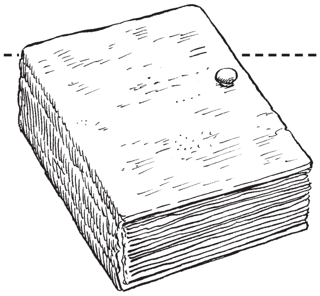
Infinite Secrets



PROGRAM OVERVIEW

NOVA explores the ongoing efforts of scientists to restore the 1,000-year-old Archimedes' Palimpsest.

The program:

- defines a palimpsest: a text in which the original text has been erased and the pages written on again.
- describes the discovery of the Palimpsest and its importance as a text that contains Archimedes' previously unknown mathematical treatises and illustrates Archimedes' process of discovery.
- explores Archimedes' life and some of his inventions, including his weapons of war.
- relates how Archimedes has become famous as the man shouting "Eureka!" in the bath when he determined how to measure volume through water displacement.
- documents Archimedes' prowess as a mathematician by providing examples of some of his key ideas, such as his methods for determining the volume of an object and estimating the value of pi, his discoveries of complex mathematical shapes and the concept of buoyancy, and his work with infinity.
- chronicles the history of the manuscript from Archimedes' time to the present and details how the Palimpsest was created—twelfth century monks in possession of the book erased the earlier recordings of Archimedes work and reused the pages for a prayer book.
- illustrates how scientists are revealing the manuscript's concealed treasures.
- concludes with speculation about how much further the study of mathematics might be if the manuscript had not been lost for a millennium.

BEFORE WATCHING

1 Archimedes made a number of mathematical and scientific discoveries during his lifetime. As they watch, have students take notes on Archimedes' key discoveries, how he made those discoveries, and the importance of each discovery.

2 Develop a timeline of important mathematicians, including those from non-Western civilizations such as Egypt, China, India, and Mesoamerica. Have students research the time and place where the mathematician lived and the contribution the person made to mathematics. Develop a class timeline and add Archimedes to place him in the historical context of other mathematicians.

3 Ask students how math is important in their everyday life. Have them give examples of how they use it or how their parents use it (such as baking cookies, paying bills, building an object, or planning a garden).

AFTER WATCHING

1 Have students refer to the notes they took about Archimedes' scientific and mathematical contributions. What were some of his inventions? Which of his discoveries were the most revolutionary and why?

2 An anonymous collector paid $2 million for Archimedes' Palimpsest (a palimpsest is a manuscript that has been written on more than once). Discuss with students what makes the book so valuable. How was it created? How was it analyzed when first found in 1906? How is it being analyzed today? What was the effect of losing the manuscript for so long?

CLASSROOM ACTIVITY

Objective
To duplicate the method Archimedes used to estimate the value of pi.

Materials for each group
- copy of the "Archimedes' Recipe for Pi" student handout
- paper
- pencil
- compass
- ruler
- calculator

Procedure
1 Tell students that they will be exploring Archimedes' method for estimating the value of pi, a mathematical constant that is the ratio of a circle's circumference (the distance around a circle) to its diameter (the distance across a circle through its center). The Greek symbol for pi is π.

2 Organize students into groups of three or four. Provide copies of the "Archimedes' Recipe for Pi" student handout and other materials to each group.

3 Define some terms for students: perimeter, circumference, radius, diameter, and area. (See Activity Answer on page 3 for more information.)

4 Demonstrate how to draw polygons that are inscribed in a circle and circumscribed around a circle:
- Draw a circle on the blackboard.
- Use a ruler to draw four lines that divide the circle into eight equal parts, extending the lines beyond the boundary of the circle.
- Connect the points where the lines meet the inside of the circle to create an octagon.
- Connect the lines around outside of the circle to create another octagon that just touches the edge of the circle.

5 Point out that the sum of the perimeters of the polygons gives an approximate value for the circumference of the circle. Students can divide each polygon perimeter by the diameter of the circle to find an approximate value for pi. If necessary, remind students that they can measure one side of a polygon, and multiply that length by the number of sides in the polygon to find the perimeter of the polygon.

6 Have the students complete the data table and find the approximate values of pi for each of the three suggested sets of polygons (square, octagon, and hexadecagon) described on the activity sheet.

7 When students have finished the activity, discuss their results using the questions on the activity sheet.

8 As an extension, have students develop fact sheets about pi.

STANDARDS CONNECTION

The "Archimedes' Recipe for Pi" activity aligns with the following Principles and Standards for School Mathematics.

GRADES 6–8
Mathematics Standard:
Geometry
Mathematics Standard:
Measurement

GRADES 9–12
Mathematics Standard:
Geometry
Mathematics Standard:
Measurement

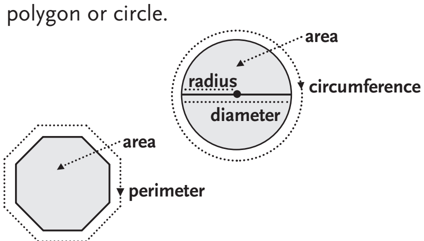
Video is not required for this activity.

Classroom Activity Author
Margy Kuntz has written and edited educational materials for 20 years. She has authored numerous educational supplements, basal text materials, and trade books in science, math, computers, art, and literature.

ACTIVITY ANSWER

If students need guidance, you may want to clarify the following concepts:

perimeter: the distance around a polygon; the perimeter is determined by the sum of the lengths of the sides of the polygon.
circumference: the distance around a circle; the circumference is the perimeter of a circle.
radius: any segment from the center of a circle to its edge.
diameter: any segment from one side of a circle to the other through the circle's center; the diameter is the same length as two radii.
area: the amount of space included within a polygon or circle.



For any circle, if you divide the circle's circumference by the diameter you will always get the same number, pi. Pi represents the ratio of the circumference of a circle to its diameter. Pi can be used to find the circumference and the area of a circle, if you know what the radius (r) of the circle is. The equations for determining those values are:

Circumference = 2 x π x r
Area = π x r x r

The concept of pi has fascinated mathematicians for more than 4,000 years, ever since people first noticed that the ratio of circumference to diameter was the same for all circles, regardless of the circle's size. Although earlier estimates of the value of pi exist, Archimedes seems to have carried out the first theoretical calculation of the constant. His approach consisted of inscribing and circumscribing regular, many-sided polygons in and around the circle, and computing the perimeters of these polygons. This provided him with the approximation 223/71 < π < 22/7, or 3.1408 < π < 3.1428. (The actual value of pi to four decimal places is 3.1415.) Pi is an infinite decimal; its value is currently known to more than 1 trillion decimal places.

Archimedes did not have access to the modern-day tools of algebra, trigonometry, or even decimal notation. Instead, he performed his calculations using purely geometrical methods. Thus, constructing and calculating the values of the perimeters of 96-sided polygons, and then using these values to estimate pi, was by no means a trivial task.

Students' answers should reflect that Archimedes' method gives an approximate range of values for pi, with the value from the inside polygon providing the lower boundary and the value from the outside polygon providing the upper boundary. In addition, students should discover that the more sides a polygon has, the better approximation of pi it provides. This is because a polygon with many sides gives a better approximation of the circumference of the circle than one with fewer sides.

Students may wonder why it was so difficult to measure the circumference of a circle. The reason is that at the time Archimedes lived, there was no way to accurately measure curved lines—only straight lines could be measured. This is why Archimedes had to devise a way to approximate a circle's circumference.

LINKS & BOOKS

Links

NOVA Web Site—Infinite Secrets
www.pbs.org/nova/archimedes/
In this companion Web site for the NOVA program, explore what infinity really means, see pages from other great surviving manuscripts, follow the journey of the Archimedes manuscript, and discover how Archimedes estimated pi.

Archimedes Home Page
www.mcs.drexel.edu/~crorres/Archimedes/contents.html
Includes information on Archimedes' life and work as well as illustrations of his inventions.

Book

Stein, Sherman.
Archimedes: What Did He Do Besides Cry Eureka?
Washington, DC: The Mathematical Association of America, 1999.
Describes the life of Archimedes, the discovery of his manuscript in 1906, and his methods for figuring out many of the concepts he developed.

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PARK FOUNDATION
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Polygon Measurements: Sample Results

Polygon Name	# of Sides	Length of Side (in cm)		Perimeter of Polygon (= number of sides x length of 1 side)		Diameter of Circle (in cm)	Value of Pi (=perimeter/diameter)	
		inside polygon	outside polygon	inside polygon	outside polygon		inside polygon	outside polygon
Square	4	2.09	3.03	8.36	12.12	3.00	2.78	4.04
Octagon	8	1.14	1.25	9.12	10.00	3.00	3.04	3.33
Hexadecagon	16	0.58	0.61	9.28	9.76	3.00	3.09	3.25

Archimedes' Recipe for Pi

One of Archimedes' many mathematical accomplishments was his computation of pi, which is the ratio of the circumference of a circle to its diameter. In this activity, you will duplicate the method he used to arrive at his estimate.

Procedure

1 Construct a data table on a sep rate piece of paper that contains the headings shown in the table below.

2 Use your compass to draw three circles on another piece of paper. Each circle can be a different size, but each should be at least 2.4 inches (6 centimeters) across.

3 Use a ruler to divide one circle into four equal pie-shape pieces. Be sure to extend your lines outside the circle. Then, using the ruler, create a square by drawing straight lines inside the circle to connect the points where the lines meet the circle.

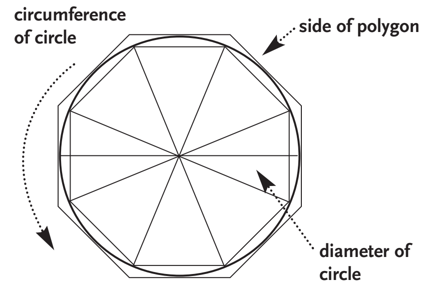
4 Connect the lines around the outside of the circle to create a second square that just touches the circle's outside edge. Make sure that the straight line for each segment touches the circle at the segment's halfway point.

5 Measure one side of the inside square. Multiply that length by the number of sides in the square (four) to find the perimeter of the inside square. Record your results in the table. Repeat the process for the outside square.

6 Use the ruler to find the diameter of the circle and record this measurement.

7 The perimeters of the squares give approximate values for the circumference of the circle. Determine the value of pi by dividing the length of each perimeter by the diameter of the circle. Record your results for both the inside and outside squares.

8 Repeat the process for the second circle, using octagons (eight-sided polygons) instead of squares. Make eight equal pie-shape pieces. Then repeat the process again for the third circle, using hexadecagons (16-sided polygons).



Questions

Write your answers on a separate piece of paper.

1 The actual value of pi to four decimal places is 3.1415. Compare the range of values you found for each set of polygons to this number. Do all three ranges include the actual value of pi? Which type of polygon gave the most accurate range of values?

2 Archimedes calculated the value of pi for polygons containing 96 sides. Do you think his calculations were more or less accurate than yours? Explain.

Polygon Name	# of Sides	Length of Side (in cm)		Perimeter of Polygon (= number of sides x length of 1 side)		Diameter of Circle (in cm)	Value of Pi (=perimeter/diameter)	
		inside polygon	outside polygon	inside polygon	outside polygon		inside polygon	outside polygon
Square	4							
Octagon	8							
Hexadecagon	16							